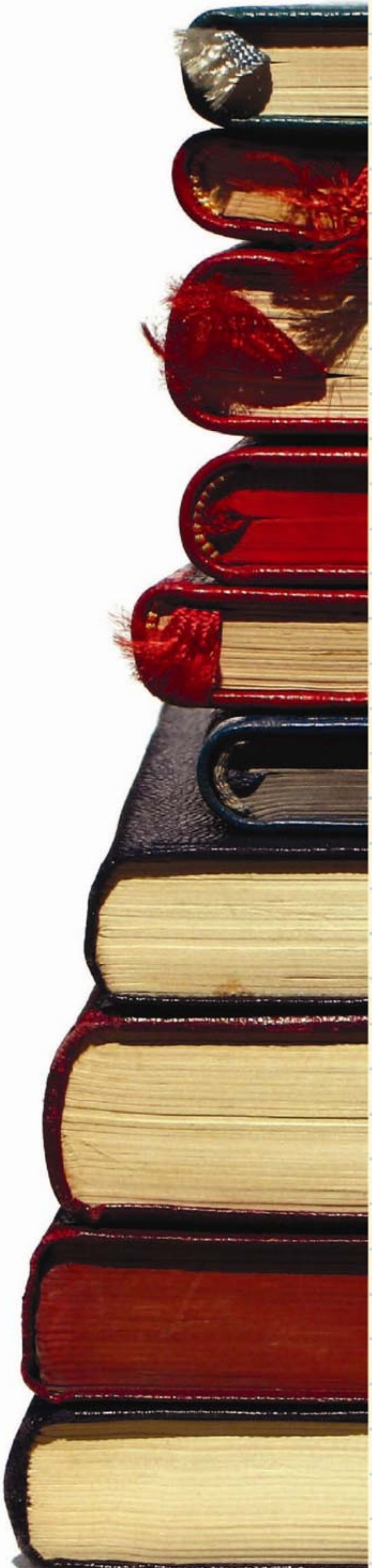

Comprehensive Curriculum

Revised 2008

Algebra I
Part 2

Louisiana Department of
EDUCATION
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Algebra I–Part 2

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***Louisiana Comprehensive Curriculum*, Revised 2008**
Course Introduction

The Louisiana Department of Education issued the *Comprehensive Curriculum* in 2005. The curriculum has been revised based on teacher feedback, an external review by a team of content experts from outside the state, and input from course writers. As in the first edition, the *Louisiana Comprehensive Curriculum*, revised 2008 is aligned with state content standards, as defined by Grade-Level Expectations (GLEs), and organized into coherent, time-bound units with sample activities and classroom assessments to guide teaching and learning. The order of the units ensures that all GLEs to be tested are addressed prior to the administration of *i*LEAP assessments.

District Implementation Guidelines

Local districts are responsible for implementation and monitoring of the *Louisiana Comprehensive Curriculum* and have been delegated the responsibility to decide if

- units are to be taught in the order presented
- substitutions of equivalent activities are allowed
- GLES can be adequately addressed using fewer activities than presented
- permitted changes are to be made at the district, school, or teacher level

Districts have been requested to inform teachers of decisions made.

Implementation of Activities in the Classroom

Incorporation of activities into lesson plans is critical to the successful implementation of the Louisiana Comprehensive Curriculum. Lesson plans should be designed to introduce students to one or more of the activities, to provide background information and follow-up, and to prepare students for success in mastering the Grade-Level Expectations associated with the activities. Lesson plans should address individual needs of students and should include processes for re-teaching concepts or skills for students who need additional instruction. Appropriate accommodations must be made for students with disabilities.

New Features

Content Area Literacy Strategies are an integral part of approximately one-third of the activities. Strategy names are italicized. The link (view literacy strategy descriptions) opens a document containing detailed descriptions and examples of the literacy strategies. This document can also be accessed directly at http://www.louisianaschools.net/lde/uploads/11056.doc.

A *Materials List* is provided for each activity and *Blackline Masters (BLMs)* are provided to assist in the delivery of activities or to assess student learning. A separate Blackline Master document is provided for each course.

The *Access Guide to the Comprehensive Curriculum* is an online database of suggested strategies, accommodations, assistive technology, and assessment options that may provide greater access to the curriculum activities**.** The *Access Guide* will be piloted during the 2008-2009 school year in Grades 4 and 8, with other grades to be added over time. Click on the *Access Guide* icon found on the first page of each unit or by going directly to the url http://mconn.doe.state.la.us/accessguide/default.aspx.

Algebra I–Part 2
Unit 1: Equations and Systems of Equations

Time Frame: Approximately six weeks

Unit Description

In this unit, linear equations and systems of linear equations in the plane are reviewed with a focus on graphing and interpreting the graphs of such equations from the standpoint of their slopes and intercepts.

Student Understandings

Students represent and find the solution to systems of two and three linear equations using graphical, substitution, and matrix methods (by calculator for systems of three equations). In the case of linear equations, students develop the graphical and symbolic methods of determining the solutions, including matrices.

Guiding Questions

1. Can students solve equations and perform operations with rational numbers?
2. Can students use the order of operations to evaluate algebraic expressions?
3. Can students write the equation of a linear function given appropriate information to determine slope and intercept?
4. Can students use the basic methods for writing the equation of a line (two-point, slope-intercept, point-slope, and standard form)?
5. Can students discuss the meanings of slope and intercepts in the context of an application problem?
6. Can students explain the meaning of a solution to a system of equations?
7. Can students determine the solution to a system of two or three linear equations by graphing, substitution, and matrix methods?
8. Can students relate the solution, or lack of solution, to a system of equations to the slopes of the lines?
9. Can students identify parallel and intersecting lines by their slopes and relate this to possible solutions?
10. Can students identify coincident lines by their slopes and *y*-intercepts and relate this to possible solutions?
11. Can students use matrices and matrix methods by hand and calculator to solve systems of equations $\mathbf{A}x = \mathbf{B}$ as $x = \mathbf{A}^{-1}\mathbf{B}$?
12. Can students solve and graph inequalities in one or two variables?

Unit 1 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Number and Number Relations	
2.	Evaluate and write numerical expressions involving integer exponents (N-2-H)
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
16.	Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
24.	Graph a line when the slope and a point or when two points are known (G-3-H)
Patterns, Relations, and Functions	
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
40.	Explain how the graph of a linear function changes as the coefficients or constants are changed in the function's symbolic representation (P-4-H)
Grade 10	
Algebra	
6.	Write the equation of a line parallel or perpendicular to a given line through a specific point (A-3-H) (G-3-H)

Sample Activities

Activity 1: Evaluating Expressions (GLEs: 9^{th} – 2, 5, 8)

Materials List: paper, pencil, math textbook, scientific calculators

Review using the order of operations to evaluate and simplify numerical expressions with exponents and parentheses. Be sure to include expressions with rational numbers. Some examples of the types of problems that students should be able to do are shown below:

Ex: $80 \div 4 \bullet 2 - 2 \bullet 2$

Ex: $4^2 \bullet 2 + [7 - (3^2 - 5)]$

Ex: Given a = $\frac{3}{8}$ and b = $\frac{-2}{5}$ determine the value of $a^2 - b^2$.

Provide additional practice problems as necessary (either through the use of textbook or teacher-made worksheets). Include in the discussion of this topic the proper use of calculators and how the expressions must be entered differently, depending on the calculator used, to arrive at the correct answer. Make sure that students use logic, estimation, and mental math strategies throughout the course when using a calculator to catch errors and make sure their answers make sense.

Activity 2: Solving Linear Equations (GLEs: 9^{th} – 5, 8)

Materials List: paper, pencil, math textbook

Review how to solve algebraic equations for an unknown value. Use this opportunity to review basic operations with adding, subtracting, multiplying, and dividing integers in the context of solving more complex equations. Discuss the order of operations as a means of solving such equations. Provide students with additional practice problems as necessary using the math textbook. Have students write a math *learning log* entry (view literacy strategy descriptions) describing in detail how they would explain to someone who had never solved an equation, the steps involved in solving linear equations with variables on both sides of the equal sign. A *learning log* is a learning strategy which forces students to put into words what they know or do not know. It offers the learner the chance to reflect on his/her understanding which can lead to further study or alternative learning paths. Students should record their *learning log* entries in a special notebook or in a separate section of their regular math notebook.

Activity 3: Graphing Lines from Points (GLEs: 9th – 13, 15, 23, 24)

Materials List: paper, pencil, graph paper, math textbook

As a teacher-led activity, write the following equation: $y = 2x - 3$ on the board or overhead. Review how to construct a two-column t-chart by making a list of (x, y) which satisfies the given equation. For example, list the following *x* values as shown below:

x	y
-2	
-1	
0	
1	
2	

Have students determine the corresponding *y*-values (–7, –5, –3, –1, 1) for each of the *x*-values shown in the chart. On the overhead, and with students following along individually at their desks, plot the points $(-2, -7)$ and $(2, 1)$ on a coordinate grid. Connect these two points with a straight line. Have the students identify the locations of the other points in the table on the graph. They should see that the other points are located along the line that was just drawn. Furthermore, students need to see that once any two points for a line are known, the corresponding line represents ALL possible solutions for the equation. Discuss how to determine the *x*- and *y*-intercepts for any linear equation or graph (i.e., to determine the *x*-intercept, let $y = 0$ and solve for *x*; to determine the *y*-intercept, let $x = 0$ and solve for *y)*. Students should also see that the *y*- intercept can be thought of as the "initial value," since it is the value of *y* when *x* is zero. Provide students with additional practice in drawing graphs of linear equations by finding ordered pairs that satisfy the equations and by finding *x*- and *y*-intercepts.

Activity 4: Slope of a Line (GLEs: 9th – 13, 23, 24)

Materials List: paper, pencil, math textbook, Using Slope BLM

This activity is designed to provide a review on the concept and use of slope in linear equations and graphs. To begin the activity, ask students to work in pairs to list everything they remember about slope from their previous math classes. Ask each of the pairs to write one item from their lists to hand to the teacher. Pick up each written item from each pair of students. Read each item written by the students and present it to the class. Ask the students to determine if each item is true or not true about slope and why. A class discussion should take place regarding each item. This procedure of eliciting a student's prior knowledge is a modified form of an *opinionaire* (view literacy strategy descriptions). An *opinionaire* is a literacy strategy designed to promote deep and meaningful understandings of the content by activating and building relevant prior knowledge and building interest in and motivation to learn more about particular topics. A critical discussion about the topic or issue follows.

Once an item is agreed upon by the entire class, add that information to a class list that the teacher writes on the board summarizing what students know about slope. Students should remember the following important ideas relating to slope:

- Slope refers to the steepness of the graph.
- Slope can be calculated by finding the *rise over run*.
- Slope is the change in *y*-coordinates divided by the change in *x*-coordinates (slope formula).
- Slope can be thought of as a “rate of change” in real-life situations (such as the slope of a distance/time graph) and the rate has both a number and a unit associated with it.

If students don’t list the formula for calculating slope, then use guiding questions to assist them in recalling it and remembering its derivation. The formula is directly related to the concept of rise over run. To find the rise, the *y*-coordinates are subtracted. To find the run, the *x*-coordinates are subtracted. The slope is the ratio of these two differences.

After a full discussion on slope has taken place, ask students to find the slope of the line using any two points generated using the equation $y = 2x - 3$ in Activity 3. Confirm that each group calculated the slope to be 2, and ask students which two points they used to determine this value. Help students realize that no matter which two points are chosen, the slope is the same. Refer students to the original equation $y = 2x - 3$. Ask if they remember the name for the form of the equation and why it is given that name. Students should see that the form is the *slope-intercept form* and that the slope and *y*-intercept are easily identifiable for a linear equation when in this form.

Remind students that the format $y = mx + b$ is very useful in graphing equations. Review the meaning of *m* and *b* in the equation. Remind students that when a point and a slope for a line are known, the graph for the line can be produced. Demonstrate on the overhead how to use slope and *y*-intercept to graph a linear equation. Plot the *y*-intercept of $\left(0, -3\right)$ on a coordinate grid. Since the slope is $\frac{2}{1}$, other points can be found on the graph by “rising” 2 units and “running” 1 unit from the *y*-intercept. [Provide additional opportunities for student to become proficient at finding slope given two points, an equation, or a graph. They should also be able to graph a line given one point and a slope.] Make copies of Using Slope BLM and discuss the worksheet after students have completed the problems.

Activity 5: Comparing Lines (GLEs: 9^{th} – 13, 23, 24, 38, 39, 40)

Materials List: paper, pencil, graph paper, graphing calculator

The goal of this activity is to get students to compare and contrast linear functions algebraically and graphically in terms of their rates of change and intercepts. It is also important that students are able to explain how the graph of a linear function changes as the coefficients or constants are changed in a linear equation.

On the same set of axes have students graph the lines $y = 2x - 2$ and $y = 2x - 3$. When graphing, have students use paper and pencil techniques first, and then use graphing calculator technology while doing this activity. Have students discuss the similarities and differences between the two graphs. The similarity is that both graphs have the same slope; the difference is there are different *y*-intercepts. Remind students that lines with the same slope are parallel lines and will never intersect. Connect the graphs with their respective equations, and point out that since both equations are in slope-intercept form, the same information could be determined by looking at *m* and *b*.

Next, have students graph the equations $y = 2x - 2$ and $y = -2x - 2$. Ask students to discuss the similarities and differences between these two graphs. In this case, the similarities are the graphs have the same *y*-intercept $(0, -2)$; the difference is the graphs have different slopes—one is positive and one is negative. Students need to make the connection that the same *y*-intercept implies a common point. Therefore, the lines will intersect at $(0, -2)$.

Finally, have students start with the equations y=2x-2 and y=x+2. Ask students to make a prediction of the similarities and differences among these two graphs without graphing. Students should write their explanation for their reasoning in words. This type of writing is a form of *SPAWN writing* (view literacy strategy descriptions). *SPAWN* is a strategy which uses higher-level thinking prompts to elicit student writing. Each letter in SPAWN stands for a particular writing prompt. They are as follows:

> S—Special Powers: students are given special powers to do things and then write about how they would use these powers.
> P—Problem Solving: students write about the solution of a problem and how they would do this.
> A—Alternative Viewpoints: students put themselves in the place of someone or something and write about it.
> W—What If?: students are asked to write on what if something happened or changed.
> N—Next: students are asked to write on what they think will happen next.

In this problem, students are being asked to write about an Alternative Viewpoint (the A in the *SPAWN* acronym) where students are putting themselves in the place of someone or something. Here they are putting themselves in the place of the two equations and determining what will be the same or different about them. Students should write their predictions on paper. Students should then graph the two equations to see if their predictions are correct. In this example, there are no similarities (except for the fact that they both have a positive slope and both are lines). The two graphs have different slopes (2 and 1), and different *y*-intercepts [$(0,-2)$, $(0,2)$].

Have the students do more comparisons on graphs, and discuss what effect each of the coefficients has on a linear graph by providing equations with varying slopes and the same *y*-intercepts.

Activity 6: Equations of Lines (Three Forms) (GLEs: 9^{th} – <u>11</u>, <u>13</u>, 15)

Materials List: paper, pencil, math textbook, Equations of Lines BLM

This activity has two main goals. The first goal is to have students determine a linear equation given a slope and a point or given two points. The second goal is to have students become proficient at translating an equation from one form to the next (i.e., slope-intercept form to standard form; standard form to slope-intercept form). Because of the content involved in this activity, several days of class time can be expected to be utilized in order to fully complete the activity.

Begin by reviewing the three forms used for equations of lines: $y-y_1=m(x-x_1)$ or point-slope form; $y=mx+b$ or slope-intercept form; and $Ax+By=C$ or standard form. Students may need to be reminded of how to find the equation of a line using a point and a slope. For example, for the point $(4,2)$ and slope = 3, ask students to generate the point-slope form for the equation. Allow students the opportunity to collaborate in finding the equation of the line in point-slope form. Once students have come up with an answer, challenge them to write this equation in the other two forms and then demonstrate how all three forms can be developed. In this example we have the following:

Point-slope form: $y-2=3(x-4)$

Slope-intercept form: $y=3x-10$

Standard form: $3x-y=10$

Do several more examples with students, including problems which involve being given two points instead of a point and a slope. Explain that in order to determine the equation of the line, the slope must be found first, and then the slope and one of the points can be used to determine the linear equation.

Next, provide students with copies of the Equations of Lines BLM. Allow students to work in small groups on the BLM, then discuss the results as a class. Assign additional problems from the textbook you are using on this topic.

Complete the activities by having students write an explanation for the following situation: "Mary just found the equation for the line that passed through the point (0, 3) and had a slope of -2. *What if* she used the slope of 2 rather than -2, how would this have changed her equation and the resulting graph? Explain your answer thoroughly." This is another example of *SPAWN writing* (view literacy strategy descriptions). In this case, students are being asked to write on "What If" (the W in the *SPAWN* acronym) something happened or changed in the problem. Having students write about mathematical ideas in words forces them students to grasp the concept more fully. In this writing, students should explain that the equation would be $y = 2x + 3$ rather than $y = -2x + 3$. The resulting graph would have had a positive slope rather than a negative slope because of her error.

Activity 7: More with Writing Equations of Lines (GLEs: 9^{th}–13, 15; 10^{th}–6)

Materials List: paper, pencil, math textbook

In this activity, extend students' understanding of writing equations of lines using a point and a slope or two points to include writing of equations of lines parallel or lines perpendicular to another line that passes through a specific point. Remind students that parallel lines have the same slope while perpendicular lines have the "negative reciprocal" slopes. Three example problems are listed below. Use these examples to discuss this topic, and include additional examples of your own as needed.

> Example 1: Write an equation in slope-intercept form of a line which is parallel to $y = 3x - 7$ and has a y-intercept of 4.
> Solution: $y = 3x + 4$
>
> Example 2: Write an equation in slope-intercept form for the line that contains the point (4,5) and that is perpendicular to the line $2x + 3y = 7$.
> Solution: $y = \frac{3}{2}x - 1$
>
> Example 3: Water freezes at 32°F and 0°C. Water boils at 212°F and 100°C. Write a linear equation which shows the relationship between the Fahrenheit and Celsius temperatures. Let F be the dependent variable and C be the independent variable. Write the equation in slope-intercept form.
> Solution: $F = \frac{9}{5}C + 32$

Have students do additional work with this skill by working similar problems found in their math textbooks.

Activity 8: Solving Systems of Equations (GLEs: 9th – 9, 11, 15, 16)

Materials List: paper, pencil, graph paper, graphing calculators, math textbook, Internet (optional)

This activity is designed to provide a review on solving a system of equations using graphing, substitution, and elimination. Provide students with the following system of equations: $x + 2y = 12$ and $2x - 2y = 6$. Review with students how to determine the point of intersection for the two linear equations using graphing, substitution, and elimination with paper/pencil methods. Include in the presentation how this can also be done using graphing calculator technology.

It is important to start this series of skills by first having students solve the problems graphically. Graphing the two lines on graph paper and on the graphing calculator shows students what they are really finding—a point of intersection between two individual lines. The other two methods (substitution and elimination) are used to produce a more accurate mathematical solution since it may be impossible to be sure of the exact location of the point of intersection graphically. Allow students to try a couple of these types of problems on their own or in pairs to provide guided practice on this skill.

Next, present the following real-life situation for students to solve:

> *James bought 3 bags of plant food and 4 plants at the nursery for a total cost of $39.00. At the same nursery, Karen bought 5 bags of plant food and 7 plants for a cost of $67.00.*

Have students work in groups of two or three to identify variables for the problem and write a system of equations using the information. Have students solve the system using the three ways discussed previously. Discuss the work after students have been given the opportunity to work in groups. (*Solution: The bags of plant food cost $5.00 per bag while the plants cost $6.00 per plant.*)

Provide other systems to solve, with both general and real-life situations, and have a discussion as to which of the three methods is best to use based on the system to be solved (e.g., $y = 2x + 3$ and $4x - 3y = 12$ might best be solved using substitution since one equation is already solved for y). Include in the examples equations which are consistent (one point of intersection), inconsistent (no points of intersection—connect this with the slope being the same or parallel lines), and dependent (where the equations are really the same and have infinitely many points of intersections). Follow this lesson with additional practice for students using a teacher-created worksheet or the math textbook. A website for a great application problem on supply and demand is: http://illuminations.nctm.org/LessonDetail.aspx?id=L382.

Activity 9: Matrices (GLEs: 9, 11, 16)

Materials List: paper, pencil, graphing calculator, math textbook

Matrices should have been fully discussed in Algebra I, Part I (see Unit 8). Review with students the way in which matrices can be used to solve systems of equations in two and three variables using a graphing calculator. Include both general and real-life situations. For example, have students write the system of equations for the following real-life situation:

> *Tickets to the school play cost $1.50 for general admission or $1.00 with a student ID card. On opening night, 325 tickets were sold for total receipts of $380. How many of each kind of ticket were sold?*

After students have written the system of equations for the situation, show students how this system could be solved using matrices via the graphing calculator. Provide additional examples using the math textbook as necessary for students to become proficient at this skill. Include systems with three unknowns. Help students to understand that geometrically, this is equivalent to essentially finding the line of intersection where three planes intersect.

Activity 10: Solving and Graphing Linear Inequalities in One Variable (GLEs: 9^{th} – 9, 11)

Materials List: paper, pencil, math textbook

Review with students how to write, solve, and graph linear inequalities in one variable. Relate solving inequalities with solving equations. Make sure students understand that the only real difference between a linear equation and a linear inequality is that in an equation there is only one solution, whereas in an inequality situation, there could be a multitude of answers or solutions. Provide examples with inequalities using $>$, $<$, $\geq$, and $\leq$ symbols. Have students model and solve real-life inequalities. For example, present the following problem:

> *Carlos makes $15 per day plus an additional $45 for each refrigerator he sells. Carlos wants his income to meet or exceed $195 per day. How many refrigerators must he sell each day?*

Have students write an inequality to express the situation and then solve the problem, graph the solution, and explain in real-world terms what the solution represents. Provide additional practice for students by incorporating material from the textbook you are using or a teacher-made worksheet.

Solution: The inequality should look something like: $15 + 45x \geq 195$, where x represents the number of refrigerators. The solution for the inequality would be: $x \geq 4$, meaning that Carlos would have to sell at least 4 refrigerators to meet his goal and any refrigerators sold past this amount would exceed his goal of obtaining an income of $195. It is also important to note that in the case of this graph, in terms of real-life solutions, there would only be whole number answers as solutions since you cannot sell a *fractional part of a refrigerator. Teaching students to make sense of the answers they find and asking questions such as, "Does this answer make sense?" is ultimately the kind of math student we want to produce…not strictly a person who can compute with no real sense of what he/she is doing.*

Activity 11: Real-World Linear Inequalities in Two Variables (GLEs: 9th – 9, 11)

Materials List: paper, pencil, math textbook, graphing calculators

Students should have learned in Algebra I Part I (in Unit 8) how to graph the solution to a linear inequality in two variables. Review the methods and shading used to express the solution to such inequalities. Since this particular topic is at the end of Unit 8, this may not be a review. If so, go through the skills required to graph linear inequalities with the care required to assure student understanding. If students can graph a line, the shading part is really the only extra skill that will need to be taught. Use a math textbook as a resource for this activity. Student should be able to use both paper/pencil graphing techniques and graphing calculator technology.

One possible example to use: Pose a situation about the different lengths of TV advertising during a 60-minute show. For example, a 60-minute TV show typically has no more than 20 minutes (or 1200 seconds) of advertising, with some ads being 30 seconds and some being 1 minute (or 60 seconds). Ask, "If ads are sold in 30-second or 1-minute blocks, how many of each type could be sold?" Have students set up and graph a linear inequality in two variables to represent this problem, and discuss the graph as a class.

Solution: If x represents the number of 30 second commercials, and y represents the number of 1 minute commercials, then the inequality $30x + 60y \le 1200$ *could be used to represent this situation. The equivalent expression* $x + 2y \le 40$ *could also be used*. Check student graphs. Since *x* and *y* actually represent the number of commercials, there are only discrete answers to this inequality. Discuss this with the class. Provide additional examples for students using the math textbook, and also incorporate the use of the graphing calculator using the "shade" function.

Sample Assessments

Performance assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The teacher will provide the student with the following information: $P = (2, -1)$ and slope = 3. The student will find three additional points for the graph and graph the line.
- The student will interpret the slope and *y*-intercept of a given real-world linear equation.
- The student will solve a system using all three techniques (graphing, substitution, and elimination).
- The student will find an example of an application of a linear equation (graph) and prepare a short paragraph on what his/her selected example means.
- The student will create portfolios containing samples of his/her activities.
- The student will take pencil/paper tests created by the teacher on the skills presented throughout this unit.

Activity-Specific Assessments

- <u>Activity 1</u>: The teacher will provide the student with an equation that has been solved incorrectly (show the steps) and the student will determine where the error occurred.

- <u>Activity 2</u>: The student will use the order of operations to evaluate a complex numerical expression containing exponents, parentheses, and rational numbers.

- <u>Activity 4</u>: The student will determine the slope of a line given a graph, two points, an equation in slope-intercept form, or an equation in standard form.

- <u>Activity 5</u>: The student will match the graphs of lines with his/her correct equations using what he/she knows about slope and y-intercepts.

- <u>Activity 7</u>: The student will determine the equation of the line given two points or a slope and a point. He/she will also write the equation of a line which is parallel/perpendicular to another line through a given point when given the equation of the original line.

Algebra I–Part 2
Unit 2: Data Analysis

Time Frame: Approximately two weeks

Unit Description

Students interpret, summarize, draw conclusions, and make predictions from data presented in tables, charts, and different graphs. There is an emphasis on using analytical skills to determine cause-and-effect relationships among data and variables.

Student Understandings

Students will organize data in tables, charts, and graphs in such a way that conclusions and predictions can be made. They will also find the equation for the line of best fit for data graphed on a scatter plot.

Guiding Questions

1. Can students organize and display data using frequency distributions, charts and tables, line plots, stem-and-leaf plots, box-and-whisker plots, and Venn diagrams?
2. Can students calculate mean, median, mode, and range, and recognize which measure of central tendency is most appropriate for a given set of data?
3. Can students interpret, summarize, draw conclusions, and make predictions using a set of experimental data presented in a table or graph?
4. Can students determine a line of best fit for a set of data?

Unit 2 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Data Analysis, Probability, and Discrete Math	
27.	Determine the most appropriate measure of central tendency for a set of data based on its distribution (D-1-H)
28.	Identify trends in data and support conclusions by using distribution characteristics such as patterns, clusters, and outliers (D-1-H) (D-6-H) (D-7-H)
29.	Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)

Grade 10	
Algebra	
5.	Write the equation for a line of best fit for a set of 2-variable real-life data presented in a table or scatter plot form, with or without technology (A-2-H) (D-2-H)
Data Analysis, Probability, and Discrete Math	
22.	Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)
23.	Draw and justify conclusions based on the use of logic (e.g., conditional statements, converse, inverse, contrapositive) (D-8-H) (G-6-H) (N-7-H)

Sample Activities

Activity 1: Measures of Central Tendency (GLEs: 9^{th}–27; 10^{th}–22)

Materials List: paper, pencil, math textbook, calculators, Frequency Table BLM

Provide students with data from a variety of real-life data sets presented in tables, line or bar graphs. Show students how to organize data using a frequency table and also using stem-and-leaf plots, and then have students determine measures of central tendency (mean, median, mode, and range) for each data set. Discuss which measure of central tendency is most appropriate for the given situation. An example of the types of problems and questions that students should be able to successfully answer are provided on Frequency Table BLM which accompanies this activity. Provide students with a variety of problems that involve measures of central tendency and/or ranges. Use a math textbook as a resource for this additional work on problems dealing with mean, median, mode, and range. Include activities in which students have to collect data themselves. After this activity have students create *vocabulary cards* (view literacy strategy descriptions) explaining how to determine the mean, median, mode, and range for a set of numbers. The use of *vocabulary cards* can help students see connections and critical attributes associated with the words. [The target word (mode) is written in the center of a card and the definition, characteristics, examples, and an illustration (if appropriate) are written in the four corners.] A possible *vocabulary card* for the word *mode* is illustrated below:

Definition Value that occurs most frequently in a set of data	*Characteristics* A set of data can have no mode, a single mode, or more than one mode
MODE	
Examples The mode for the set of data below is 44. 43, 34, 44, 35, 44, 21 There is no mode for this set of data: 34, 43, 45, 46, 47	*Illustrations* N/A

Activity 2: Trends in Data (GLEs: 9th–27, 28; 10th–22)

Materials List: paper, pencil, How Much Does a Bag of Apples Weigh? BLM

In this activity, discuss with students how to identify trends in data by looking at patterns, clusters, and outliers. Provide copies of How Much Does a Bag of Apples Weigh BLM that accompanies this activity. In this worksheet, students organize the list of the weights of bags of apples into a stem-and-leaf plot. Students then determine the mean, median, mode, and range for the weights of the apples. Students should notice that although the weight indicated on each bag is supposed to be three pounds, the actual weight of each bag may not be 3 pounds but instead represents the "average weight" of a bag of apples.

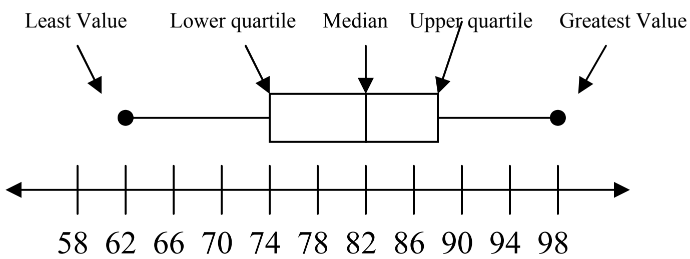
Activity 3: Box-and-Whisker Plots (GLEs: 9th–27, 28; 10th–22)

Materials List: paper, pencil, Internet, graphing calculator

Review with students what a box-and-whisker plot is, how to create it, and how to interpret the graph. When dealing with large amounts of data and graphing the results, sometimes too much data can be hard to summarize. The advantage of a box-and-whisker plot is that it gives the person interpreting the data a good idea of how the data is distributed, or spread, and how symmetric the distribution of the data is by graphing only five values for the data set—the minimum value; the maximum value; the median; the lower quartile; and the upper quartile.
Discuss with students how to find these values. The lower quartile value is found by determining the median value of the lower half of the data. The upper quartile is found by determining the median of the upper half of the data. The difference between the upper quartile and the lower quartile is called the inter-quartile range. This provides another measure of the variation in the data.

Teacher note: There are various methods of calculating the upper and lower quartiles for a set of data. The Department of Education will NOT use the median of a data set when calculating the upper/lower quartile if there is an odd number of data items. Example: 2, 5, 7, 9, 12 Median is 7. When the upper quartile is found, the median will be found by finding the average of 9 and 12 which is 10.5, eliminating 7 from the calculation of either the upper or lower quartile calculation.

An example of a box-and-whisker plot is shown below.



The least and greatest values are marked with a dot. The lower quartile, median, and upper quartile are connected to form a "box," and the least and greatest value points are connected with a line or "whisker." In this example, showing the grade distribution for a math class, students should be able to interpret the results as follows: 50% of the students scored between 74 and 88 on the test; the median grade was an 82; the lowest grade was a 62; the highest grade was a 98; 50% of the students scored higher than an 82. Provide additional examples and opportunities for students to graph and interpret box-and-whisker plots using both paper and pencil and graphing calculator technology.

There is also an excellent internet link which allows the user to investigate the mean, median, and their relationship to a box-and-whisker plot for a set of data that they create at http://illuminations.nctm.org/ActivityDetail.aspx?ID=160.

Still another website that has real life data sets already graphed on box-and-whisker plots as well as allowing the users to input their own data is the site located at http://illuminations.nctm.org/ActivityDetail.aspx?ID=77.

Activity 4: Venn Diagrams (GLE: 10th—22, 23)

Materials List: paper, pencil

Still another way to display and organize data is through the use of Venn diagrams. Students should have been exposed to the use of such diagrams for several years prior to this course. It is being included here as skill maintenance from earlier grades. Review with students how Venn diagrams are used and how they can help solve problems. Present the following situation to students.

At Oakview Academy there are 125 tenth grade students. The tenth grade students were surveyed to determine their favorite food so that the lunch menu could be adjusted. The results of the survey were as follows:

79 of the students surveyed liked pizza
61 of the students surveyed liked hamburgers
22 of the students surveyed liked salads
6 of the students surveyed only liked pizza and salads
5 of the students surveyed only liked salads and hamburgers
26 of the students surveyed only liked pizza and hamburgers
4 of the students surveyed liked pizza, hamburgers, and salad

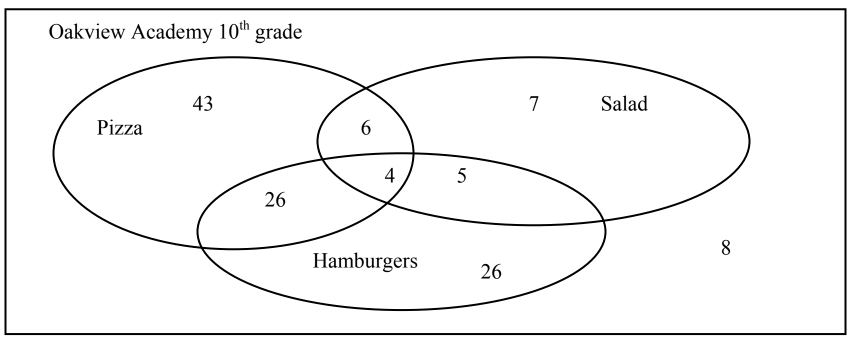
Using this data, have the students answer the following questions (it should be noted that not every one of the 125 tenth grade students were present the day the survey was taken):

(1) Use a Venn diagram to determine how many of the 125 students were present at school the day the survey was taken and how many were absent.
Solution: 117 people took part in the survey; 8 people were absent.

(2) How many people surveyed only liked Pizza?
Solution: 43 students liked pizza but not salads or hamburgers.

(3) How many people surveyed only liked Salads?
Solution: 7 students liked salads but not pizza or hamburgers.

(4) How many people surveyed only liked Hamburgers?
Solution: 26 students liked hamburgers but not pizza or salads.



At the end of this activity, have students create a *SPAWN writing* (view literacy strategy descriptions) using the "P" from SPAWN as the writing prompt. In this particular case, have students explain the strategy they used as they solved this problem and what they think is the best approach or strategy to use when solving a problem using a Venn diagram. Students should realize that the key to solving these types of problems is working from the inside-out, filling in what the intersections are and then seeing what is left to complete the Venn diagram. Provide additional problems for students to become proficient at problem solving using Venn diagrams.

Activity 5: Line of Best Fit (GLEs: 9th –29; 10th – 5, 22)

Materials List: paper, pencil, graphing calculators, grid paper, math textbooks, Internet

In this activity, the website http://illuminations.nctm.org/ActivityDetail.aspx?ID=146 will be utilized to show students how to use a scatter plot to display the real-life data associated with the number of minutes played by basketball players and the number of points they scored. When accessing the website, open up the *Instructions* and *Explorations* tabs to access the data that is to be entered. In this activity, the user is allowed to enter a set of data, plot the data on a coordinate grid, and determine the equation for a line of best fit. After working on this activity via the computer, have students plot the data using graph paper and model the process of determining a line of best fit using paper/pencil methods. Use a straightedge to draw the line so that the line best matches the overall trend. Help students understand how to determine the equation for the line of best fit using the slope and *y*-intercept of the line drawn. Afterwards, demonstrate how to make a scatter plot and find a line of best fit for the data using graphing calculator technology.

Provide students with additional opportunities to work with real data. For example, have students plot the diameter in relationship to the circumference of round objects they measure, and then determine the equation for the line of best fit for the data. Problems of this type can be found in most math textbooks. Utilize the textbook to provide additional work and learning experiences concerning the use of scatter plots and determining line of best fit for a set of data.

Activity 6: Exploring Linear Data (GLEs: 9th – 28, 29; 10th – 5, 22)

Materials List: paper, pencil, graphing calculators, grid paper, Internet, copies of Activity Worksheets (found at the website listed below).

In this activity, students model linear data in a variety of settings that range from car repair costs to sports to medicine. Students work to construct scatter plots, interpret data points and trends, and investigate the notion of line of best fit. Students then determine the equation for the line of best fit and use it to interpret real-life situations. The activities are located at the following site: http://illuminations.nctm.org/LessonDetail.aspx?ID=L298. This site includes all information needed for this lesson, including extension activities and assessment ideas.

Activity 7: Interpreting Data from Graphs (GLEs: 9th–28; 10th–22)

Materials List: paper, pencil, internet, computer spreadsheet graphing utility (optional), graph paper

Provide students opportunities to evaluate and create different types of graphs including bar, circle, and line graphs. A spreadsheet program may be used to create the graphs.

Given a situation and data from a table, have students determine which graph would be most appropriate to display the data. Have students write a math *learning log* (view literacy strategy descriptions) explaining why they chose the type of display for each situation, and then construct, label, and scale the graph. Have students analyze and interpret each graph. Some examples are provided below. Additional examples can be found at the website: http://mathforum.org/workshops/usi/dataproject/index.html.

Data Table 1: Candy Sales for a School Fundraiser

Number of Candy Bars Sold	375	280	240	400
Week #	1	2	3	4

Data Table 2: Amount of Water Poured into a Bathtub over Time

Minutes	0	1.5	3.5	5.5
Gallons of Water	0	22.5	52.5	82.5

Data Table 3: Student Expenses

Expenses	Food	Rent	Entertainment	Clothes	Books	Other
% of Total	30%	25%	15%	10%	10%	10%

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

General Assessments

- The student will write a paper on the history of statistics in mathematics.
- The student will do a project where he/she collects real-life data, represents the data using an appropriate graph, and then interprets the results of the data that was collected.
- The student will solve a problem using a Venn diagram.

Activity-Specific Assessments

- Activity 1: The student will determine the measures of central tendency for use in reporting the "average" of different types of data (i.e., average grade, average salary for a given profession, average height of adult males or females) and then select the measure that is best suited for that data set.

- Activity 2: The student will take a set of data and organize the data into a stem-and-leaf plot and frequency table. Students find the mean, median, mode, and range of the data and determine which measure of central tendency best represents the data. Also have students determine any outliers and clusters in the set of data.

- Activity 3: The student will be provided a data set and asked to create a box-and-whisker plot. The student will then analyze and interpret the graph.

Algebra I–Part 2
Unit 3: Probability and Odds

Time Frame: Approximately three weeks

Unit Description

Students study the relationships between experimental and theoretical probabilities. This unit focuses on examining probability through simulations as well as the use of odds. There is an emphasis on using advanced methods of determining the nature of possible outcomes and representing the results.

Student Understandings

Students will use counting and grouping methods in permutation (with and without replacement) and combination problems. In addition, students will understand the relationship between finding the probability of an event occurring and the odds of an event and be able to determine one if given the other.

Guiding Questions

1. Can students create simulations to approximate the probabilities of simple and conditional events?
2. Can students relate the probabilities associated with experimental and theoretical probability analyses and express these probabilities as percents, decimals, and fractions?
3. Can students use areas of figures and geometry to determine the probability of an event?
4. Can students create lists and tree diagrams to generate combinations and sample spaces?
5. Can students handle permutation problems with repetitions allowed and more advanced combination contexts?
6. Can students relate probabilities of events to the odds associated with those events?

Unit 3 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
9th Grade	
Number and Number Relations	
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Data Analysis, Probability, and Discrete Math	
30.	Use simulations to estimate probabilities (D-3-H) (D-5-H)
31.	Define probability in terms of sample spaces, outcomes, and events (D-4-H)
32.	Compute probabilities using geometric models and basic counting techniques such as combinations and permutations (D-4-H)
33.	Explain the relationship between the probability of an event occurring, and the odds of an event occurring and compute one given the other (D-4-H)
10th Grade	
Data Analysis, Probability, and Discrete Math	
21.	Determine the probability of conditional and multiple events, including mutually and non-mutually exclusive events (D-4-H) (D-5-H)
24.	Use counting procedures and techniques to solve real-life problems (D-9-H)

Sample Activities

Activity 1: Using Lists and Tree Diagrams (GLE: 10th–24)

Materials List: paper, pencil, calculators, The Counting Principle BLM

Review with students how to create lists and tree diagrams in order to graphically show the possible arrangements in a given situation. Lists and tree diagrams will later be used to determine the sample space for a particular event in order to determine probability.

Present students with various real-life situations involving counting the number of ways an event can occur. For example, suppose Jim has 6 shirts and 5 pants, and he wants to figure out how many different outfits he can make out of the shirts and pants. Assuming all of the shirts and pants are different from one another, call the shirts (A, B, C, D, E, F) and the pants (1, 2, 3, 4, 5) and create a tree diagram or list such as the one shown below.

A-1	B-1	C-1	D-1	E-1	F-1	
A-2	B-2	C-2	D-2	E-2	F-2	
A-3	B-3	C-3	D-3	E-3	F-3	6 rows x 5 per row = 30 outfits
A-4	B-4	C-4	D-4	E-4	F-4	
A-5	B-5	C-5	D-5	E-5	F-5	

Do several of these types of problems with students, then connect this with *The Counting Principle*. The counting principle is used to find the total number of ways in which two or more events can occur, and it is calculated by finding the product of the ways the individual events can happen. In this example, there are 6 shirts and 5 pants. Since there are 6 ways to pick a shirt, and each shirt can be paired with any of the 5 pants, using the counting principle produces the same result as the listing technique: 6 x 5 = 30 different outfits altogether.

Include problems such as “If there are 8 people in line, how many ways could they be lined up in a single line for a picture?” In this case, students should be lead through a discussion that would sound something like this: In the first slot, there are 8 possible choices of people to fill the spot; after filling the first slot, there are 7 people left to fill the second slot; after filling the first two slots, there are 6 people left to fill the third slot, and so on. Ultimately, students should understand that determining the total number of ways to arrange 8 people for a picture can be found by multiplying: 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 40320 ways. (Discuss the use of the factorial key at this point and how to use the calculator to do the calculation 8!). When a full discussion of counting techniques has taken place, provide students copies of The Counting Principle BLM, and let the students work in groups on the problems then discuss the answers as a class. Provide students with additional work on these types of problems from a math textbook or other resources which contain this topic.

Activity 2: Permutations (GLE: 9^{th} – 32; 10^{th}–24)

Materials List: paper, pencil, calculators, Permutations BLM

Begin the activity by distributing copies of the Permutations BLM to students. Discuss the student note at the top of this BLM, and explain how the counting principle will be used to solve permutation problems (counting problems where order is important) as discussed on the worksheet. As a class, work the first two problems from the BLM and discuss the solutions thoroughly. Next, let students work in groups to solve the remaining problems, and then discuss them as a class. When a thorough discussion has taken place on the BLM, assign additional permutation problems using a math textbook as a resource.

Activity 3: Combinations (GLE: 9^{th} – 32; 10^{th}–24)

Materials List: paper, pencil, calculators, Combinations BLM

Begin the activity by distributing copies of the Combinations BLM to students. Discuss the student note at the top of this BLM which explains the difference between problems where order is not important (combinations) with those where order is important (permutations). Discuss this difference, and then work the first two problems presented on the BLM together as a class.

> Note to Teacher: If students have a hard time understanding why they have to divide by the number of arrangements to find the total number of combinations, have students write out all 3 letter orderings for the pizzas. Then, have students cross out those orderings that are the same. While this method takes time, it helps students to understand why they must divide, and they aren’t just memorizing a formula.

Next, let students work in groups on solving the remainder of the problems and then discuss them as a class. When a thorough discussion has taken place on all the problems from the BLM, assign additional problems using a math textbook or other material as a resource. Be sure to provide problems in which students must determine if a combination or permutation is to be used. Finally, have students write a math *learning log* (view literacy strategy descriptions) entry explaining the difference between a permutation problem and a combination problem along with an example of each type of problem of their own making. Check these to make sure students understand the unique difference between the two types of problems.

Activity 4: Sample Spaces and Simple Probability (GLEs: 9th–5, 31; 10th–24)

Materials List: paper, pencil, calculators

Review with students the meaning of probability. Ask students if they remember how to determine the theoretical probability of an event occurring. Students should remember that probability is defined as:

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

For example, in a simple probability situation such as tossing a die, the probability of tossing an even number is 3 out of 6 or $\frac{3}{6}$ or $\frac{1}{2}$ since there are 6 sides possible and 3 sides that have an even number. Ask students if they can give a range of values for the probability of an event based on the definition. Try to get students to think. They should be able to relate this to a weather man's forecast for snow, rain, etc. Students should understand that probability can range from 0 (no chance of an event's occurring) to 1 (the event will always occur). Point out to students that probability can be expressed as a fraction, a decimal, or as a percent.

In more complex probability situations, creating a sample space to show the total possible outcomes can help in finding the probability of an event. A sample space is essentially the same thing as making a list or drawing a tree diagram that lists all the ways an event can occur. For example, suppose there are 4 meats (ham, salami, pastrami, and roast beef) and 3 breads (rye, wheat, and white) to choose from at a deli. If a person goes into the deli and orders a sandwich consisting of one type of meat and one type of bread, what is the probability that the person chooses salami on rye? Creating a sample space can help show the total possibilities (12), and since one of them is salami on rye, the probability the person will choose such a sandwich would be $\frac{1}{12}$ or .083 or 8.3%. Provide students with a broad array of simple probability problems whereby the students must make a sample space to help find the probability. Provide additional work for students using a math textbook or other materials.

Activity 5: More Complex Probability (GLEs: 9th–5, 31, 32; 10th–24)

Materials List: paper, pencil, calculators, More Complex Probability BLM

Provide students with copies of More Complex Probability BLM. Discuss how the counting techniques learned earlier can be used to find the total possible outcomes in a more complex probability problem. Discuss the first two problems from the BLM, then allow students to work in groups on the remaining problems. Include in the initial discussion, the use of the notation, P(A). This is simply a shorthand way of asking the learner to find the "probability of event A" occurring.

Discuss the BLM thoroughly after students have completed it. Afterwards, have students do *SPAWN* writing (view literacy strategy descriptions) describing the way in which they solved one of the problems from the BLM. This writing focuses on "P," the Problem Solving category of the *SPAWN* literacy strategy. When students have completed their writing, have students exchange their papers with a partner to provide feedback on its accuracy and logic. For additional practice, find more problems for students to try working complex probability problems from a math textbook or some other resource.

Activity 6: Probability Experiments (GLEs: 9th–5, 30, 31, 32; 10th–24)

Materials List: paper, pencil, calculators, two-color counters, paper bags, paper cups

Show students a two-color counter and ask students to determine the theoretical probability of the counter landing on red. Students should agree that the probability of it landing on red is ½ or 50%. Explain to students that there are two types of probability: theoretical (which they have found using the mathematical definition) and experimental probability (which is based on simulations or experiments). Place the students into groups and provide each group with a generous quantity of two-color counters, a small paper bag, paper, and pencil. Ask each group to place a single counter into the paper bag, shake it vigorously, and dump the counter onto the tabletop. Have one student from each group go to the board or overhead and fill in its results on a chart. The red side of the counter likely will appear about the same number of times as the yellow. Indicate to students that they performed a simulation or experiment one time and ask them to find P(red) for the one time experiment based on the data the class collected. Theoretically, when determining the probability of flipping the two-color counter and getting red, $\frac{1}{2}$ or 50% of the time the expected outcome would be a red. However, in real life, this may or may not occur. Compare the experimental results collected by the students to the theoretical probability.

Next, have each pair of students do the experiment 10 times, and see how many times red shows up in each group. Compare the results as a class. Students might be surprised to see that the red side may or may not occur five out of ten times. Finally, find the total results for the whole class—this should be closer to what is theoretically expected. Talk about the *Law of Large Numbers* which states that the experimental results get closer and closer to the theoretical expectations the more times the experiment is done.

Provide additional experiments for students to learn more about experimental probability. Below is another idea for having students determine experimental probability:

Paper Cup Experiment:
Provide each student a paper cup. If you toss the cup there are three possible outcomes:

1) the cup could land standing right side up
2) the cup could land upside down
3) the cup could land on its side

Have students make a guess as to what they think the probability of each outcome might be and write them as a percent below:

P(right side up) = _________ P(upside down) = _________ P(side) = _______

Have students actually toss the cup 100 times and keep a tally of the results then express the results as the experimental probability. Have students compare the experimental results with the guesses they made and ask them to explain if they were surprised at the results. Ask them to find the theoretical probabilities and compare these with the experimental results.

Activity 7: Determining Probability Based on Sample Data (GLEs: 9th–5, 30, 31, 32; 10th–24)

Materials List: paper, pencil, calculators, Probability Based on Sample Data BLM,

Discuss with students that in real-life, the probability of some events is difficult or even impossible to determine. What makes them difficult is the sheer number of data items that are involved. In this case, samples are taken and the probabilities are based upon the samples of a given situation or population. Use the following problem as an example to discuss with students: A company wanted to determine the probability that the product it manufactures is damaged upon leaving the plant that produces it. This company produces 1000 radios per day. Rather than testing all radios, the company tests 20 radios out of each 1000 produced. If one of the 20 randomly chosen radios is defective, the company estimates that the same proportion of radios would be defective for each 1000 radios produced. Since $\frac{1}{20}$ is 5%, the company expects that 5% of the 1000, or 50 radios, would be defective.

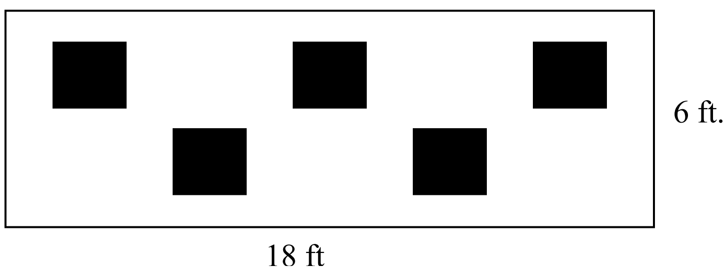
Provide students copies of the Probability Based on Sample Data BLM, and allow students the opportunity to work with a partner on the problems. Discuss the BLM after all students have completed the work. Provide students additional practice on these types of problems using a math textbook or a teacher-created worksheet.

Activity 8: Geometric Probability (GLEs: 9th–5, 31, 32; 10th– 24)

Materials List: paper, pencil, calculators

Suppose a game of darts is being played and the player wants to determine the probability of hitting a specific ring on the board. What if a coin is thrown on a square board on which there are smaller black squares and a player wants to know the probability that the coin will land on one of the black squares? Situations such as these can be modeled using geometric figures. As result, this method is sometime referred to as "geometric probability."

Have students determine the geometric probability for various problems using a teacher-created worksheet or problems from a math textbook or some other resource. For example, suppose a person throws a coin onto the board shown below. If the coin is equally likely to land anywhere on the board, what is the probability of the coin's landing in a shaded square if the shaded squares are 2 feet by 2 feet? Answer: $\frac{20}{108}$ or $\frac{5}{27}$ or .185 or 18.5%.



You may also want to pull some problems from the following website which has several problems that deal with geometric probability: http://jwilson.coe.uga.edu/emt668/EMT668.Folders.F97/Hales/lessons/Lesson%205/Lesson5.html. Another website allows the user to randomly throw darts at a target of various areas to determine the experimental probability using geometry: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=59.

Activity 9: What are the Odds? (GLEs: 9th–5, 33)

Materials List: paper, pencil

Inform students that another method of describing the likelihood of an event occurring is called "finding the odds" of an event. Explain to the students that in order to determine the *odds* in favor of an event, we must determine the ratio of the number of ways an event can occur to the ways the event cannot occur.

Ask the students to create the sample space describing the outcomes of tossing two coins (*heads-heads, heads-tails, tails-heads, and tails-tails*). Have the class decide how many ways two heads can be obtained from the experiment (*Answer: 1 way*). Ask the class to decide how many ways something other that two heads can be a result (*Answer: 3 ways*).

Explain to the class that this would mean that the *odds* of getting two heads when flipping two coins would be 1 to 3 or 1:3 or $\frac{1}{3}$. (Note: clear up any confusion students may have at this point when writing ratios as fractions; i.e., $\frac{1}{3}$ in this case does not mean that there are 2 heads in $\frac{1}{3}$ of the cases…stress this to students).

Ask the class to determine the *probability* of getting two heads ($\frac{1}{4}$) and compare that number to the *odds* of getting two heads. Students should understand that the odds of getting two heads are related to the probability of getting two heads in the following way: The odds being 1:3 tells us that there are actually four outcomes with 1 being two heads and 3 of the outcomes not being two heads. Since probability is ¼ we can see that 1 of the four possible outcomes is two heads. We want students to recognize the relationship between the odds of an event occurring and the probability of that event occurring in such a way as to be able to find the odds, given the probability, and finding the probability, given the odds.

Provide additional practice by using the experiment of rolling one number cube. Ask the students to find the odds of rolling a 3 on a single die (*1:5*), rolling a 3 or a 6 on a single die (2:4 or 1:2), or a 2, 3, 5, or 6 on a single die (*4:2 or 2:1*). Provide students with various problems that involve finding the odds, and have them compare this to the probability of an event occurring. For example, suppose the probability of someone's winning a prize is $\frac{2}{13}$, what are the odds of winning the game? (*Answer: 2:11*). Use a math textbook as a resource for additional problems.

Have students create *vocabulary cards* (view literacy strategy descriptions) for finding the odds of an event and the probability for an event. When students create these *vocabulary cards*, they should be shared with the class and students should provide feedback to one another on the accuracy of the cards that are created. These cards can then be used by students to help in preparing for major tests or end of unit exams. An example of a *vocabulary card* for the term probability is provided below:

Definition The likelihood that an event will occur		*Characteristics* Probability is measured on a scale from 0 to 1 or 0% to 100% There are two types of probability: experimental and theoretical
	Probability	
Examples Probability is defined as: (number of favorable outcomes) (total number of outcomes)		*Illustrations* P(1 Head flipping 2 coins) = ½ HH, HT, TH, TT

Activity 10: Independent Events (GLEs: 9th–31; 10th–21, 24)

Materials List: paper, pencil, calculators

Have students determine the sample space for rolling two different colored die—students should see that there are 36 different outcomes. Ask the students to determine the probability of rolling a 4 on the first die and a 3 on the second die [noted as *P*(4,3) using the sample space they created.] Students should see that the probability $P(4,3) = \frac{1}{36}$. The fact that a 4 was rolled on the first die has absolutely no effect on the 3 being rolled on the second die. In other words, the probability of getting either of these two outcomes has nothing to do with one another. This type of event is referred to as a *Independent* or *Mutually-Exclusive* event. Two events, A and B, are independent if the probability of B does not depend on what happened in A. To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. This multiplication rule is defined symbolically below. Note that multiplication is represented by AND: P (A and B) = P (A) · P (B). Discuss how this problem could have been found as follows: $P(4) \times P(3)$ since the two events are independent of one another. Since $P(4) = \frac{1}{6}$ and $P(3) = \frac{1}{6}$, then the $P(4,3) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. In order to use this principle, the events have to be independent of one another. Provide students with more examples using a math textbook or some other resource, and have students solve real-life probability problems using this concept. Another great resource for problems of this type can be located at the following site: http://www.mathgoodies.com/lessons/vol6/independent_events.html.

Activity 11: Compound Events (GLEs: 9th– 30, 31; 10th–21, 24)

Materials List: paper, pencil, calculators, 7 blue and 3 red objects for each pair of students (use paper squares or cubes)

Have students perform an experiment whereby they place 7 blue and 3 red objects in a bag and then find the experimental probability of pulling a blue object on the first pull and a red object on the second pull (replacing the first object pulled before pulling the second object from the bag). Let students investigate this problem experimentally (100 pulls) to see what the results were for each group then what the class averaged as a whole.

Next, show students how this problem could have been solved theoretically. This problem involves what is referred to as a compound event. In this particular compound event, the two events are mutually exclusive or independent of one another—the second pull does not depend on the outcome of the first pull. To find the probability of pulling a blue marble, followed by pulling a red marble, P(B,R), students must recognize that the events are independent because the marble drawn first is replaced. The two individual probabilities are multiplied. $P(\text{B}) = \frac{7}{10}$ and $P(\text{R}) = \frac{3}{10}$, so $P(\text{B,R}) = \left(\frac{7}{10}\right)\left(\frac{3}{10}\right) = \frac{21}{100}$ or 21%. Compare these results with what the class determined experimentally.

Next, do the same experiment but this time without replacement (i.e., do not put back what is pulled on the first pull into the bag). The second pull is now dependent on the first pull and the second probability will now be affected. The probability of pulling blue on the first pull is still $\frac{7}{10}$, but since this blue marble will not be replaced, the container will only have 9 marbles in the second pull. Thus, the probability for pulling red on the second pull would be $\frac{3}{9}$ or $\frac{1}{3}$. The probability in this situation would be as follows: $P(\text{B,R}) = \left(\frac{7}{10}\right)\left(\frac{1}{3}\right) = \frac{7}{30}$ or 23.3% (which is slightly higher than in the other case). Have students do the problem experimentally to see how the results compare to the previous experiment. Provide ample opportunity for students to solve problems involving independent and dependent events using a math textbook as a resource for additional problems.

Activity 12: This or That! (GLEs: 9th–31, 32; 10th – 21, 24)

Materials List: paper, pencil, calculators

Have students work in pairs to generate a table of the possible sums one could get when two dice are rolled. Because the sums are 2, 3, 4, . . . 11, and 12, ask students to determine the probability of getting a sum of seven. Students should realize that $P(7) \neq \frac{1}{11}$ in order to calculate the number of ways to get each sum (i.e., 2 = 1 + 1). There are 36 possible ordered pairs to use to compute the sums. Thus, the rolls that could give a sum of 7 are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1). So $P(7) = \frac{6}{36} = \frac{1}{6}$. Now, ask students to calculate the probability of getting a sum of 9. Again, by counting the ways to get a sum of 9, students should find $P(9) = \frac{4}{36} = \frac{1}{9}$.

Now, ask students to determine the *P*(7 or 9). Because these events are independent, the probability is found by adding the individual probabilities *P*(7) and *P*(9), giving *P*(7 or 9) $= \frac{6}{36} + \frac{4}{36} = \frac{10}{36} = \frac{5}{18}$. To find the combined probability of two independent events (such as "this or that"), add their individual probabilities. Point out the difference between this situation and the situation in which the individual probabilities are multiplied. Include problems that require geometric probabilities. Use the math textbook as a resource for additional problems of this type. After this lesson, have students write a math *learning log* (view literacy strategy descriptions) entry explaining the difference between a probability problem that uses "AND" and one that uses "OR," and why it is that in one type you multiply the individual probabilities, and in the other type you add the individual probabilities. Let students explain this using sample spaces and problems they create themselves.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

General Assessments

- The student will determine whether a game is fair or unfair based upon the odds of winning and losing.
- Paper and Pencil tests of the topics listed throughout the unit determining probability, odds, combinations, permutations, sample spaces, independent and dependent probabilities.
- The teacher will ask the student to write a paragraph that compares and contrasts the meanings of the terms *probability* and *odds*. The student will discuss the relationship between the probability of an event's occurring and the odds of an event's occurring.
- The student will determine the probability of an event's happening given the odds.
- The student will develop simulations to help determine an experimental probability for a complicated set of events.

Activity-Specific Assessments

- Activity 1: The student will create a tree diagram to determine the total possible outcomes for a combination or permutation situation.
- Activity 4: The student will determine the probability of simple events and express his/her answers using fractions, percents, and decimals. The student will compare the likelihood for each event and put the probabilities in order.
- Activity 8: The student will solve real-life geometric probability problems.
- Activity 11: The student will solve probability problems with and without replacement using dependent and independent events. The teacher will make sure the problems vary in scope and complexity.

Resources

- *Facing the Odds – The Mathematics of Gambling* http://www.louisianaschools.net/lde/uploads/5924.pdf

Algebra I–Part 2
Unit 4: Angle Relationships, Right Triangles, Similarity, and Proportions

Time Frame: Approximately five weeks

Unit Description

This unit emphasizes and explores the link between geometry, measurement, algebra, and similar triangles. In addition, the meaning and mathematical nature of similarity is examined. Included is work with the Pythagorean Theorem and its applications. In addition, the study of angles in relation to parallel lines cut by a transversal and polygon angle sums is included.

Student Understandings

Students solve for the lengths of sides in similar triangles using direct proportions and apply the similarity concept to solve real-world indirect measurement problems. Students can identify and calculate the measures of angles formed by the intersection of parallel lines and a transversal.

Guiding Questions

1. Can students identify the angle relationships between the angles formed from parallel lines cut by a transversal?
2. Can students determine the angle sum of different polygons?
3. Can students correctly apply the Pythagorean Theorem to solve problems involving right triangles?
4. Can students determine whether two figures are similar to one another?
5. Can students use proportions to find the lengths of missing sides of similar triangles?

Unit 4 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
6.	Simplify and perform basic operations on numerical expressions involving radicals (e.g., $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$) (N-5-H)

GLE #	GLE Text and Benchmarks
7.	Use proportional reasoning to model and solve real-life problems in solving direct and inverse variations (N-6-H)
Measurement	
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)
22.	Solve problems using indirect measurement (M-4-H)
Grade 10	
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
4.	Use ratios and proportional reasoning to solve a variety of real-life problems including similar figures and scale drawings (N-6-H) (M-4-H)
Geometry	
9.	Construct 2- and 3- dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and *n*-gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
11.	Determine angle measurements using the properties of parallel, perpendicular, and intersecting lines in a plane (G-2-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
18.	Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence (G-5-H) (M-4-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)

Sample Activities

Activity 1: Parallel Lines and Angle Relationships (GLEs: 10^{th}–10, 11)

Materials List: paper, pencil, protractors, a math textbook, Parallel Lines Investigation BLM

Review with students by asking them what they remember about how to measure angles using a protractor. Have them recall what they know about the terms acute, right, obtuse and straight angles and the measures associated with each category of angle measure. Also review the terms complementary and supplementary in terms of angle measure. Students should remember that two angles are complementary if the sum of their measures adds to 90 degrees. Two angles are supplementary if the sum of their measures adds to 180 degrees. Discuss the terms parallel and perpendicular, and have students list some everyday places one would see parallel or perpendicular line segments.

This review should naturally lead into the Parallel Lines Investigation BLM. Provide an opportunity for students to investigate the angle relationships of parallel lines cut by a transversal. Have students use a protractor to measure and then make conjectures about angle relationships. Hand out copies of Parallel Lines Investigation BLM, and have students work in pairs on this activity. Go over the results as a class. The goal of the investigation is to allow students the opportunity to become reacquainted with their previous work on this topic. The role of the teacher here should be try to get students to remember the vocabulary used and to indicate the angle relationships. Formally discuss the names and the theorems associated with student findings such as the terms transversal, vertical angles, corresponding angles, alternate interior angles, and alternate exterior angles. Discuss the angle relationships that are always formed when a pair of parallel lines is cut by a transversal. This information should be verified through the investigation activity.

Have students create *vocabulary cards* **(**view literacy strategy descriptions) for all the geometry terms they learned during this activity. An example of a *vocabulary card* for the term alternate interior angles is shown below. Remember to have students utilize the cards when studying for tests or to let students quiz one another via the cards they created.

Definition
Two angles whose sides form opposite rays

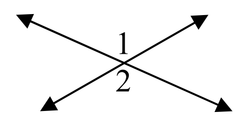
Characteristics
If two angles are vertical, they are congruent to one another

Vertical Angle

Examples
angle 1 is 40° and is vertical to angle 2, then angle 2 is also 40°.

Angle 1 and 2 are vertical angles (shown in the illustration to the right).

Illustrations

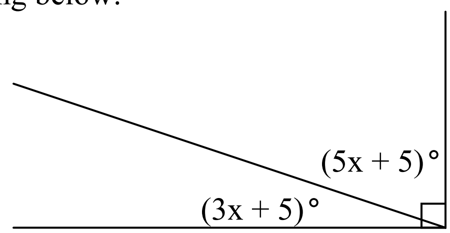


Provide students with additional work on this topic using a math textbook as a resource. For example, have students find the missing angles when one or more angles are listed in a drawing of parallel lines cut by a transversal.

Activity 2: Using Algebra in Solving Angle Measure Problems (GLEs: 10th–10, 11)

Materials List: paper, pencil, Using Algebra in Solving Angle Measure Problems BLM

Provide students with the opportunity to solve the measures of angles using what they learned in Activity 1, and include application problems with algebraic expressions and equations. Explain to students how to use algebra to solve for unknown angle measures when algebraic expressions are used to represent the measures of angles. For instance, refer to the drawing below:



In this drawing the two angles are complementary since they form a right angle. If one angle is "$3x+5$" and the complementary angle is "$5x+5$", then the equation $3x+5+5x+5=90$ can be used to solve for *x*. In this case, $x=10$, and the two angle measures would be $35°$ and $55°$, which do actually give a sum of $90°$. Provide additional examples of these types of problems relating complementary, supplementary, and vertical angles. Include work on problems which deal with parallel lines cut by a transversal. Next, provide students with copies of Using Algebra in Solving Angle Measure Problems BLM, and allow students to work in groups on the activity. Go over the results as a class. When completed, provide additional work of this type using a math textbook as a resource for additional problems.

Activity 3: The Polygon Angle-Sum Relationship (GLEs: 10th–10)

Materials List: paper, pencil, Polygon Angle-Sum Investigation BLM

Begin this activity by questioning students on information they should remember from their previous work with geometry. Include the following information during the questioning process:

- "What is the sum of the angles of any triangle?" Review with students the fact that in any triangle, the sum of the angles is $180°$.
- "What is a polygon?" Clear up any misconceptions students might have. A polygon is a closed plane figure with at least three sides that are straight line segments. The sides of a polygon intersect only at their endpoints and no adjacent sides are collinear. Perhaps drawing figures on the board and asking students which are polygons and which are not, then having students describe what attributes a polygon has would be a good starting point for the discussion.
- "What is the mathematical name of a 3, 4, 5, (etc.), sided polygon?" Students may not remember all of the names, so review with students the naming scheme we

use to name polygons with 3, 4, 5, 6, 7, 8, 9, 10, and n-sides (i.e., triangle; quadrilateral; pentagon; hexagon; heptagon; octagon; nonagon; decagon; n-gon).

Once the review of terminology has been done, have students work in pairs on the Polygon Angle-Sum Investigation BLM. The BLM is meant to be done as an investigation by students, whereby they determine a pattern and develop a formula based upon the pattern they see. The students should obtain the formula for finding the sum of the interior angles for any polygon. The Polygon Angle-Sum Theorem states that the sum, S, of the angles of an n-gon can be found by the formula: S = 180(n-2). We know that the angle sum of any triangle is 180°. If we can determine the number of triangles that a polygon contains, all we have to do is multiply the number of triangles by 180° to find the sum of the angles for the particular polygon. The formula comes from the fact that an n-gon which has n sides forms (n-2) triangles, and since each triangle has 180° we simply multiply 180(n-2) to get the sum of the interior angles of any polygon.

Provide additional work on this topic using a math textbook as a resource. Include work where there are missing angles to be found as well as work which includes the use of algebra in determining missing angles (as in Activity 2).

Have students write a math *learning log* (view literacy strategy descriptions) entry explaining where the Angle-Sum Theorem comes from in their own words and check their logs for correctness. Another option is to have students share their writing with the class. The class decides whether they agree or disagree with the writer and the writer defends his/her choice (or decides to alter it based on the review from the class).

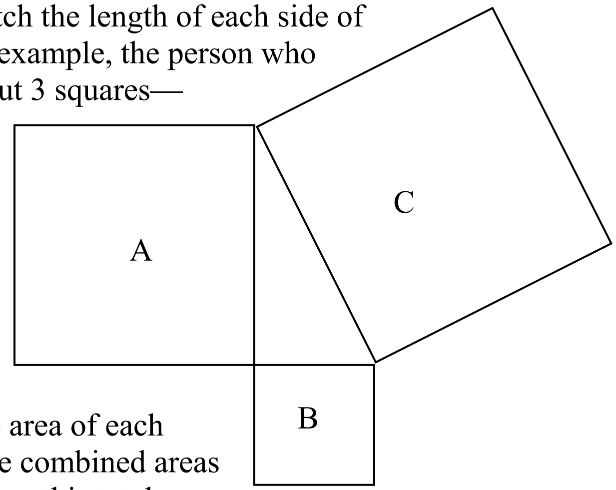
Activity 4: The Pythagorean Theorem Proof (GLEs: 10th–9, 10, 12, 19)

Materials List: paper, pencil, centimeter grid paper, scissors, protractors, rulers

While all types of triangles have been studied throughout history in math, particular attention has been paid to right triangles. In this activity, students will review the Pythagorean Theorem and do a simple proof of the theorem. Have students work in groups of 3 on the following tasks:

- First, have each student in each group construct a right triangle with different side lengths—one person should draw a right triangle with sides 3 cm, 4 cm, and 5 cm; one person should draw a right triangle with side lengths 6 cm, 8 cm, and 10 cm; and one person should draw a right triangle with side lengths 5 cm, 12 cm, and 13 cm.

- Next, have each student cut out squares from square centimeter grid paper, whereby the dimensions of the square match the length of each side of the particular triangle he/she created. For example, the person who constructs the 3, 4, 5 triangle should cut out 3 squares—one 3x3 square; one 4x4 square; and one 5x5 square.
- Next, have the students place the 3 squares created along the corresponding sides of the triangle they created as shown in the picture at right:



- Have students actually count the square units on the grid paper for each of the squares that were created to determine the area of each square formed. Have students calculate the combined areas of the two smaller squares and then compare this total area with the area for the larger square. All three students in each group should realize that the *sum of the areas of the squares on the smaller two sides equals the area of the square on the biggest side*.
- Label the smaller two legs *a* and *b* and the hypotenuse *c*. The area of the square on the *a* side is a^2; the area for the square on the *b* side is b^2; and the area for the square on the *c* side is c^2. Thus, the algebraic representation of the Pythagorean theorem, $a^2 + b^2 = c^2$, states that the *sum of the squares of the two legs is the same as the square of the hypotenuse.*

Use the following website to help students develop an understanding of the theorem: http://www.pbs.org/wgbh/nova/proof/puzzle/theorem.html. The site contains an interactive applet that allows students to see the relationship between the areas of the squares. There are also many different proofs of the theorem—have students do a report on one of the proofs and present their findings to the class.

Activity 5: Solving Pythagorean Theorem Problems (GLEs: 9th–4, 6; 10th–1, 12)

Materials List: paper, pencil, calculators, a math textbook, Internet

Now that students have seen a derivation of the Pythagorean Theorem, have them solve problems that utilize the Pythagorean Theorem. Students should understand that the theorem applies only to right triangles. Explain to students how this theorem is primarily used to determine the missing side length of a right triangle when two of the other sides are known. Demonstrate for students how this is done mathematically. A thorough review of simplifying radical expressions (discussed in Part 1 of this course) will probably be in order. Review what a *perfect square* is, how to simplify radical expressions, how to compute with radical numbers, how to approximate the value of a square root that is not a perfect square, and how calculators introduce approximations—not exact solutions—when working on irrational numbers. Provide students with a wide array of application problems dealing with the Pythagorean Theorem. Utilize a math textbook as a resource for finding application problems of this type. Also, the following website contains ten real-life application problems which utilize the Pythagorean Theorem that are of different types: http://www.regentsprep.org/Regents/math/fpyth/PracPyth.htm

Activity 6: Pythagorean Relationships (GLEs: 9th–7; 10th –12, 18)

Materials List: paper, pencil, calculators

In this activity, discuss how the Pythagorean Theorem can also be used to determine if a triangle is a right triangle. Begin by providing students with copies of triangles that are not right triangles. Have students measure the three sides of the triangles provided, then have students apply the Pythagorean Theorem to the triangles. Ask students what they notice when trying to apply the theorem to non-right triangles. Students should realize that the Pythagorean Theorem does not work with triangles which are not right triangles. The Pythagorean Theorem states that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. However, if the triangle is not right, then the side lengths will not produce the same results. Explain that this idea of using the Pythagorean Theorem to determine whether or not a triangle is right is an application of the Converse of the Pythagorean Theorem. The converse states: If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

Discuss with students how to determine whether a triangle is a right triangle or not based upon the numbers alone (without a drawing). For example, the triangle with side lengths 1, 3, 5 cannot form a right triangle since $1^2 + 3^2 = 5^2$ is not true. However, the triangle with side lengths 5, 12, 13 will form a right triangle since $5^2 + 12^2 = 13^2$. Have students verify different combinations of side lengths that will and will not form a right triangle.

Talk about the common Pythagorean triples: 3- 4 -5; 5-12-13; 7- 24 - 25; and 8 -15-17. Point out that these (especially the first two) occur and are used often in math, including the multiples of these (3-4-5 leads to 6-8-10 and 9-12-15, etc.). Show students how knowing these triples and their multiples can help them quickly determine a third side length in a right triangle if two lengths are known and are two values in one of the triples. For example, if the sides of a right triangle are given as 14 - x - 50, and if we know that the side which is 50 units is the longest side, then the missing side length is 48 since 7-24-25 doubled is 14-48-50.

Next, use this opportunity to have students compare the areas of the two right triangles (7-24-25 and 14-48-50). Students should realize that although the side lengths of the second triangle are doubled, the area for this triangle is actually four times that of the first triangle. Discuss how and why this occurs mathematically as a class.

Activity 7: What is a Similar Figure? (GLEs: 9^{th}–7; 10^{th} –4)

Materials List: paper, pencil, rulers, protractors, Similar Figures BLM, Are the Polygons Similar? BLM

This activity is to familiarize students with what it means for two figures to be similar to one another. Students should be familiar with the concept of "congruent" figures—figures that are exactly alike in every way. Congruent figures have the same size and shape. Similar figures have the same shape, but not necessarily the same size (although two congruent figures are also similar to one another).

Provide students with copies of Similar Figures BLM. Have students make measurements of the side lengths for the two rectangles using an inch ruler. Have students determine the ratio of the shorter side to the longer side for each of the two rectangles. Explain to students that in order for two figures to be considered similar figures, two conditions must be met: (1) all pairs of corresponding angles are congruent to one another; and (2) all pairs of corresponding sides must have the same ratio, called the scale factor. Therefore, in order to be similar, the ratio of the lengths of pairs of sides of one figure must be proportional to the ratio of the lengths of the pairs of corresponding sides of the other figure. Ask students to determine if the two rectangles are similar to one another given this definition. Students should realize that the two figures are similar, and the scale factor in this case is 2 since Figure 2 has side lengths twice the size of Figure 1. Help students to understand that a similar figure is nothing more than an enlargement or a reduction of a figure.

Provide students with examples of various geometric figures that are similar to one another and some which are not similar to one another, including triangles and rectangles. Make sure students get in the habit of checking all sides in order to make sure the two figures are truly similar. Discuss how to mathematically determine (using proportions) whether two figures are similar, then let students get into groups to determine if various polygons are similar using the Are the Polygons Similar? BLM. After students have attempted the BLM, go over the answers as a class and discuss what the similarity ratio is for any pairs of figures that it thinks are similar to one another.

Activity 8: Find the Missing Side Length of Similar Figures (GLEs: 9^{th}–7; 10^{th} –4, 18)

Materials List: paper, pencil, a math textbook

Teach students about the ways to write proper proportions when solving for missing side lengths for similar figures. When setting up proportions for two similar rectangles, one could write the ratios in the following ways:

Ex 1: $\frac{length1}{length2} = \frac{width1}{width2}$ Ex 2: $\frac{length1}{width1} = \frac{length2}{width2}$

Provide students with the opportunity to utilize proportions to find the missing side lengths of proportional figures such as triangles and quadrilaterals of different types (rectangle, square, rhombus, parallelogram). For example, provide students with the measures of some corresponding sides, and have them find the missing measures of the remaining sides. Include many real-world problems which utilize this concept. Use a math textbook as a resource for problems of this type. Include problems with scale drawings. Expose students to all types of applications where a scale factor and a missing length are found.

Activity 9: Indirect Measurement Problem (GLEs: 9th –4, 7, 21, 22; 10th–4, 18)

Materials List: paper, pencil, measuring tapes, meter sticks, calculators, a math textbook

In the classroom, explain that it is possible to calculate the height of the school's flagpole (or telephone pole) without actually measuring it by using the concept of similar triangles and shadow lengths. Provide students with measuring tapes and meter sticks. Have students work in groups of three to *brainstorm* (view literacy strategy descriptions) how to use similar triangles to find the height of the flagpole. Have students write a plan for solving the problem of determining the height of the flagpole, and then share ideas with the whole class in a group discussion. After the discussion, students should better understand how proportions and similarity can be used to determine the length of the flagpole.

Have the class move outside to the flagpole. Have the teams complete their plans and then return to the classroom. Remind students that all measurements must be recorded. Students are to use their real-world data to solve the problem and find the height of the flagpole. Record the answers of the groups and ask the students why all groups did not get the same solution (errors induced by measurement). Lead students to determine that one way to get a better estimate of the height might be to find the average of the heights calculated by the different groups. Follow this problem up with additional problems of this type from a math textbook.

Sample Assessments

General Guidelines

Performance assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will write a short explanation of the flagpole experiment—what was done and why it works.
- The student will write a report on the origin and proof of the Pythagorean Theorem. The student will make a presentation of his/her report using computer technology such as the Internet and power point.
- The student will interview a carpenter to find out how the Pythagorean Theorem (as well as other geometry and measurement concepts) is used in the carpentry world. The student will present his/her findings to the class or in a written report.
- Paper/pencil tests on the concepts learned during this unit.

Activity-Specific Assessments

- Activity 2: The students will solve problems in which he/she has to determine the missing angle measure of a polygon, supplementary angles, complementary angles, or angles formed by parallel lines cut by a transversal. Problems should include both numeric as well as algebraic solutions.

- Activity 5: The student will apply the Pythagorean Theorem to solve a real-life problem. The student will show all work and provide a written explanation of his/her solution.

- Activity 7: Provide the student with a worksheet with pairs of geometric figures, and have students determine if each pair of figures is congruent, similar, or neither.

- Activity 8: The student will solve problems involving finding a missing side of a figure, given a figure which is similar to it.

- Activity 9: The student will solve an application problem involving indirect measurement.

Algebra I–Part 2
Unit 5: Coordinate and Analytic Geometry

Time Frame: Approximately three weeks

Unit Description

In this section, geometric concepts involving coordinate geometry are reviewed and expanded upon. The focus is on using coordinate geometry as well as synthetic and transformational methods to solve problems.

Student Understandings

Using coordinate geometry, students will apply and verify properties of two-dimensional figures including distance, midpoint, slope, parallelism, and perpendicularity. Students will also perform transformations on the coordinate plane.

Guiding Questions

1. Can students locate a missing point on a coordinate grid that would form a particular polygon or shape?
2. Can students determine the distance and midpoint between two points on a coordinate grid and connect the distance formula with the Pythagorean theorem?
3. Can students determine the slope of a line segment that connects two points on a coordinate plane and apply this concept to use slope to verify whether lines are parallel or perpendicular?
4. Can students find transformations and mappings that relate one congruent figure in the plane to another?
5. Can students provide an argument for the determination of whether a figure has given characteristics or matches certain criteria (such as determining whether a figure on a grid is a parallelogram)?

Unit 5 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Number and Number Relations	
6.	Simplify and perform basic operations on numerical expressions involving radicals (e.g., $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$) (N-5-H)

GLE #	GLE Text and Benchmarks
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as a rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
26.	Perform translations and line reflections on the coordinate plane (G-3-H)
Grade 10	
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
14.	Develop and apply coordinate rules for translations and reflections of geometric figures (G-3-H)
15.	Draw or use other methods, including technology, to illustrate dilations of geometric figures (G-3-H)
16.	Represent and solve problems involving distance on a number line or in the plane (G-3-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)

Sample Activities

Activity 1: Using Slope to Determine Geometric Properties (9^{th}–23; 10^{th}–19)

Materials List: paper, pencil, graph paper, math textbook

Review the slope formula with students, asking them first to state what they remember about the formula. Use this as an opportunity to review whether two lines are parallel to one another (parallel lines have the same slope) or whether two line segments are perpendicular to one another (perpendicular lines have slopes which are the negative reciprocal of one another). These concepts were developed when discussing algebraic concepts, but they will now be used in geometric applications to prove figures are parallelograms, trapezoids, or other specified figures.

Discuss how slope can be used to prove whether figures have parallel or perpendicular sides. Provide students with the following coordinates, and have students graph and label the points on a coordinate grid: $A(-9,-2); B(-7,2); C(-4,3); D(-6,-1)$. Have students form groups and have them *brainstorm* (view literacy strategy descriptions) ways to use slope to determine if a figure is a parallelogram. Discuss any strategies students came up with, and decide as a class which are valid to use.

Students should determine that figure ABCD is a parallelogram because each pair of opposite sides has the same slope.

Next, provide students with the points: $A(-9,-2); K(-10,1);$ and $P(-6,-1)$. Ask students to determine what kind of triangle is formed if the points are connected by finding the slopes of the sides. Students should prove that triangle AKD is a right triangle. Segment AK has a slope of –3 and segment AD has a slope of $\frac{1}{3}$ which proves that the two sides are perpendicular to one another. Provide students with an opportunity to work with additional examples using the math textbook as a resource.

Activity 2: Finding the Missing Point (GLEs: 9th–<u>23</u>; 10th–9)

Materials List: paper, pencil, graph paper, math textbook

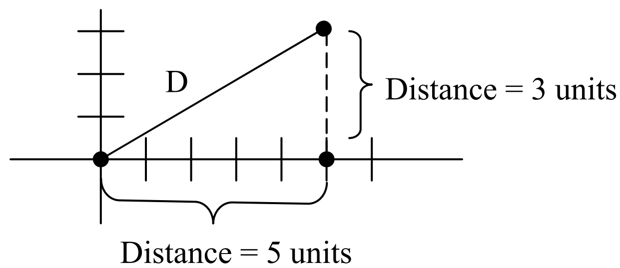
Have students locate a missing point in a coordinate grid that would create a particular 2 dimensional figure. For example, provide students with the following coordinates: $(-1,-3);$ $(-1,5);$ and $(5,-3)$. Have students locate these coordinates on a coordinate grid. Next, have students find the missing coordinate(s) needed to form a parallelogram. Students should realize that there are actually 3 possible locations for a point on the grid, which along with the three given points would form a parallelogram. The point $(5,5)$ will form a rectangle—which is a parallelogram. The other two solutions are $(-7,5)$ and $(5,-11)$. Provide additional examples for students using the math textbook as a resource.

Activity 3: Distance on a Coordinate Grid and the Pythagorean Theorem (GLEs: 9th–6, 23; 10th–<u>1</u>, <u>12</u>, <u>16</u>, 19)

Materials List: paper, pencil, graph paper, math textbook

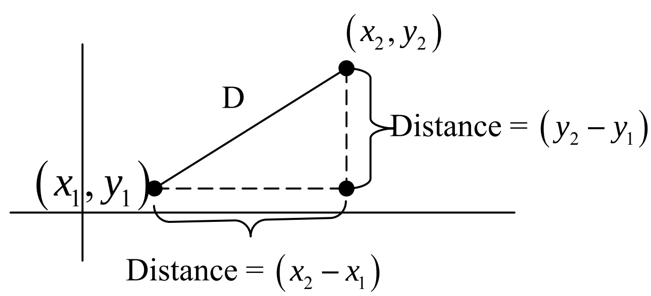
Lead a discussion with students on how to find the distance between two points on a coordinate grid. Start out with simple problems that involve counting segments along a grid such as the problem of finding the distance from the point $(0,0)$ to $(5,0)$. If the points are plotted on a coordinate grid, the distance is obviously 5 units.

Lead students to understand how to solve distance problems using the distance formula. Build on what students should already know by connecting the distance formula with the Pythagorean Theorem. For example, have students find the distance between points $(5,3)$ and $(0,0)$. Plot these points onto a coordinate grid as shown in the graph below, and then show how to find a vertex which would form a right triangle.



Since the figure forms a right triangle, and since two sides of the triangle have a distance of 3 and 5 units long, students should see that the third side, D, can be found by using the Pythagorean theorem. In this particular example, $D^2 = 5^2 + 3^2$ or if we solve for D by taking the square root of both sides we have $D = \sqrt{5^2 + 3^2}$. Thus, the distance in this example would be $D = \sqrt{34}$ or approximately 5.8 units long.

Lead students to connect this method of finding the distance between two points with the more general case found using the distance formula. If two points have the coordinates (x_1, y_1) and (x_2, y_2), the distance between the *x* coordinates is $(x_2 - x_1)$. The distance between the *y* coordinates is $(y_2 - y_1)$.



Since these differences are the lengths of the sides of a right triangle, the length of the hypotenuse, D, can be found using the equation: $D^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ or

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

After making sense of the formula, provide students with the opportunity to solve distance problems in both abstract and real-life problems using the math textbook as a resource for additional problems. For example, have students prove that two line segments are the same length if two sides of a polygon are congruent. Use this also as an opportunity to discuss how to simplify and find the approximate value of a radical. For example, if $D = \sqrt{56}$, this could also be thought of as $D = 2\sqrt{14}$.

Complete the activity by having the students create a *vocabulary card* (view literacy strategy descriptions) for the Distance Formula. Remember to allow students the opportunity to share their cards with other students and make changes based upon the feedback they get. An example of a vocabulary card is shown.

Definition $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$		*Characteristics* Used to determine the distance between two points on a coordinate grid. The formula comes from the Pythagorean Theorem.
	Distance Formula	
Examples Determine the distance between (5,3) and (2,4) $D=\sqrt{(5-2)^2+(3-4)^2}$ $D=\sqrt{9+1}$ $D=\sqrt{10}$		*Illustrations* N/A

Activity 4: Midpoint as Average (GLEs: 9th–23)

Materials List: paper, pencil, graph paper, math textbook

Discuss with students what they know or remember about a midpoint. This discussion should lead to the formal definition that a midpoint is a point which lies on the same line and is halfway between two other points.

If a software drawing program such as The Geometer's Sketchpad is available, have students draw a segment, have the software find the midpoint, and display the coordinates of the segment's endpoints and midpoint. Have students move the endpoints and record at least four sets of coordinates as the segment is moved. If the software is not available, have students plot points on a coordinate system for which the midpoints are easily determined ($(0,0)$ and $(0,4)$, (2,5) and (2, 11)) and record their results. Have students examine the coordinates for a pattern. Students should determine that the coordinates of the midpoint are the average of the *x*-values of the two points and the average of the two *y*-values of the two points. Lead students to develop an understanding of the midpoint formula:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Provide examples and use the math textbook as a resource for additional problems. Be sure to include examples in which the midpoint and one endpoint are known and have the students find the missing endpoint.

Activity 5: Problem Solving Using Coordinate Geometry (GLEs: 9th–6, 23; 10th–1, 9, 12, 16, 19)

Materials List: paper, pencil, graph paper, math textbook, Coordinate Geometry BLM

Have students solve problems that involve using what they learned about coordinate geometry basics such as distance, midpoint, and slope. Hand out copies of Coordinate Geometry BLM, and let students work on these problems in small groups. Once everyone has completed the assignment, have students share their solutions with the class. Discuss fully each problem. Find additional problems if possible for students to work on using the math textbook as a resource. Provide students with different types of problems including real-life application problems that can be solved using coordinate geometry.

Activity 6: Visualizing Transformations (Using Technology) (GLEs: 9th–26; 10th–14)

Materials List: paper, pencil, computer or computer lab, internet, Vocabulary Self-Awareness BLM

Begin this activity by having students assess their own understanding of the some of the terms that will be discussed in this activity. Provide students with the Vocabulary Self-Awareness BLM that accompanies this activity. The goal of using *vocabulary self-awareness* (view literacy strategy descriptions) is to have students become aware of what they know about a particular topic or term, as well as what they still need to learn in order to fully comprehend the subject. Words or topics are introduced at the beginning of the lesson, and students complete a self-assessment of their knowledge of the material. The chart is then used to identify the strengths and weaknesses of the students, and adjustments are made to the chart as their understanding changes.

In this particular use of the strategy, have students fill out the Vocabulary Self-Awareness BLM and then pick up their responses. Use the assessment to guide instruction, making sure to pay particular attention to the material that most students may not understand. At the end of the activity, hand out the Vocabulary Self-Awareness BLM, and have students fill out the chart once again. Students should compare their responses with what they initially knew about the topics listed. The expectation is that students see their own growth in the understanding of the topics or terms that they didn't know before the lesson.

After students have made their initial assessment of their understanding of the terms using the BLM, begin the activity by having students access http://standards.nctm.org/document/eexamples/chap6/6.4/index.htm. This *Visualizing Transformations* section allows students to visualize and investigate the various kinds of transformations using dynamic geometry software. If possible, use the school computer lab and let students work in pairs on this activity. If no school lab exists and only a one-computer classroom is available, display the results for the class via a television monitor to talk about transformations. Remember to use the self-assessment results to guide instruction. Ask questions, such as: How does the original shape compare to the shape after the transformation? What is the effect of the transformation on the side lengths and angle measures of the original shape? Discuss the results as a class.

After the introductory work is done, use http://www.utc.edu/Faculty/Christopher-Mawata/transformations/translations/lesson2.html to allow students the opportunity to further explore the effects of transformations. This site allows more interaction with the various transformations, but it also includes work on the coordinate grid. There are yellow arrow keys at the bottom of each page to scroll from one topic to the next, and there are questions to investigate for each topic. Again, if possible, allow students to work in the school lab in pairs, and then discuss the results as a class.

To conclude this activity, have students write a *learning log* (view literacy strategy descriptions) describing in words the types of transformations they learned about. Have students share their logs with one another; then let students give feedback on anything that was not clear. Allow students the opportunity to edit their logs based on the feedback they were provided. Finally, remember to have students go through the Vocabulary Self-Awareness BLM when finished with this activity to help them see their own growth in the understanding of the subject matter.

Activity 7: Transformations on a Coordinate Grid (GLEs: 9th–26; 10th –14)

Materials List: paper, pencil, Transformations on a Coordinate Grid BLM

Provide students with a copy of Transformations on a Coordinate Grid BLM. Have students work in pairs on the activity. This activity is intended as an opportunity for students to apply what they learned in Activity 6 to finding transformations on a coordinate grid. Once students have had the opportunity to do the BLM in pairs, discuss the work as a class. Afterwards, have each pair of students create their own problem of this type and then exchange with another pair of students. Allow students the chance to work on each other's problems, and then let the original writers of the problem check the other pair's work for correctness.

Activity 8: Rotational Symmetry (GLEs: 9th–26; 10th–14)

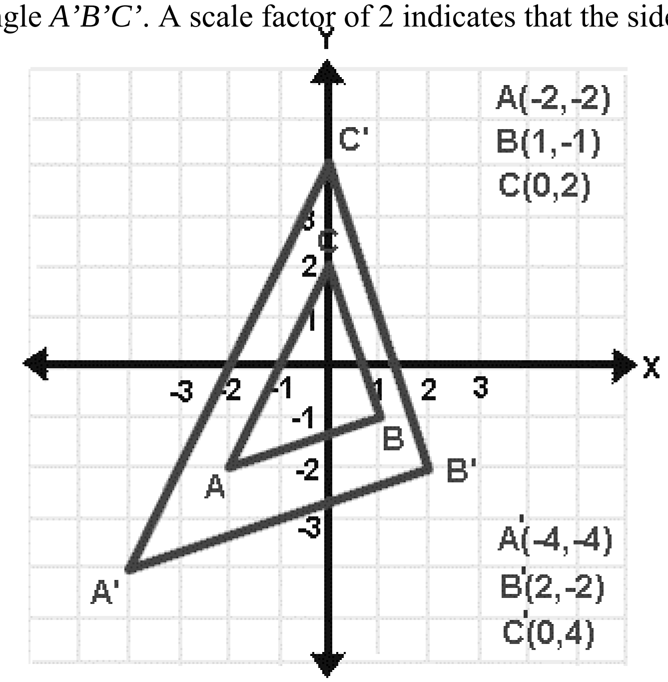
Materials List: paper, pencil, Internet, computer or computer lab

In this activity, rotational symmetry is explored. The web site for NCTM Illuminations located at http://illuminations.nctm.org/LessonDetail.aspx?id=U138 provides four lessons on rotational symmetry: describing rotations; finding what doesn't change; relating rotations to symmetry; and conclusions. The lessons contain interactive computer generated applets which allow the user to visually show how rotational symmetry works and to display rotations of any angle. The lessons contain questions, activities, and answer keys to further develop students' understanding of transformations and their applications. Students can explore these lessons in a computer lab setting, or if a one-computer classroom is the only option, lead the students through the lessons using technology available. Note: Rotations are taught in earlier grades, but this will review students on the topic.

Activity 9: Magnify It! (GLE: 10th – 15)

Materials List: graph paper, pencil, math textbook, Geometer's Sketchpad, computer

Discuss with students what dilation is. Dilation is a transformation that produces an image that is the same shape as the original, but is a different size**.** The description of dilation includes the scale factor and the center of the dilation. The example below shows how triangle *ABC* is dilated with the center of dilation at the origin with a scale factor of 2 forming triangle *A'B'C'*. A scale factor of 2 indicates that the side lengths are all doubled.



A dilation of scalar factor k whose center of dilation is the origin may be written as: $D_k(x, y) = (kx, ky)$. If the scale factor is greater than 1, the image is an enlargement. If the scale factor is between 0 and 1, the image is a reduction. Most dilations in coordinate geometry use the origin $(0,0)$ as the center of the dilation—this is simply the point at which every length will be measured from. After discussing with students what a dilation is and how to draw a dilation with a given scale factor, have students work in groups of two to develop a specific dilation of a figure that has been graphed in the plane. For example, have them create a dilation that is 1.5 times the size of the original figure. As an extension, instruct students in how to create a dilation that is .75 times the size of the original figure. Be sure to instruct students to specify the coordinates of the dilated figures. In addition to drawing the dilation by hand, students can use *The Geometer's Sketchpad* or other drawing program to perform dilations.

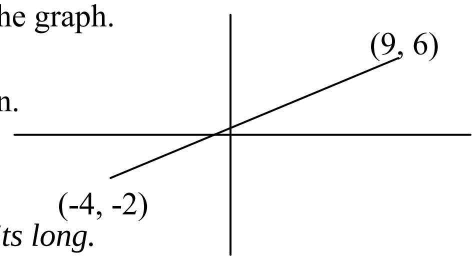
Sample Assessments

General Guidelines

Performance assessments can be used to ascertain student achievement.

General Assessments

- Provide the student with a polygon in the coordinate plane, and have him/her perform various transformations on it.
- The student will create a tessellation using glide reflections, rotations, and translations. A rubric based on the number and/or type of transformations made could be used.
- The student will develop a rotation transformation with a predetermined angle by using a pair of intersecting reflection lines.
- The student will create portfolios containing samples of his/her activities.
- Allow students to work together in groups to solve the following problem:
  a. Find the slope of the line segment shown in the graph.
  Solution: $m = \frac{8}{13}$
  b. Find the midpoint of the line segment shown.
  Solution: *Midpoint* = $\left(\frac{5}{2}, 2\right)$
  c. Find the distance between the two points.
  Solution: $D = \sqrt{233}$ *or approximately 15.3 units long.*



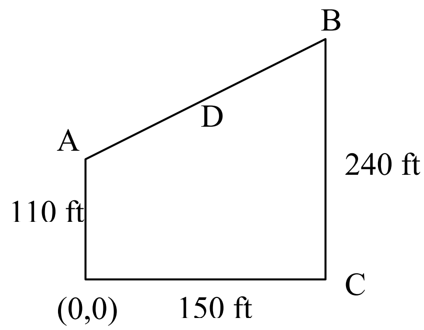
Activity-Specific Assessments

- Activity 1: The student will solve the following problem: Line segment AB has a slope of $\frac{1}{2}$. If B has coordinates $(6,10)$ and A has coordinates $(x,7)$, what is the value of x? Explain how you know?
  Solution: The value of x is 0. Explanations may vary.

- Activity 2: The student will find the missing point on a coordinate grid that will form a particular type of figure.

- Activity 3: The student will find the distance between two points on a grid using the distance formula.

- <u>Activity 5</u>: The student will solve problems involving coordinate geometry. For example, have the student solve the following problem: An obstacle course, which Larry works out on, has a 110-foot wall to climb (Point A). After climbing the wall, Larry then must propel along a rope (AB) across a waterway using his hands to reach the finish line (Point B) of the obstacle course. A picture of the course is shown below.



  - Consider the bottom of the 110-foot wall to be $(0,0)$ on a coordinate grid. What are the coordinates of A, B, and C?
    Solution: $A(0{,}110); B(150{,}240); C(150{,}0)$
  - What is the slope of the rope?
    Solution: $\frac{13}{15}$
  - What is the length of the rope?
    Solution: Approximately 198.5 feet.
  - If point D is the halfway point up the rope, what are its coordinates?
    Solution: Midpoint is $(75{,}175)$

- <u>Activity 9</u>: The student will perform a dilation on a figure in a coordinate grid with a given scale factor.

Algebra I–Part 2
Unit 6: Measurement and Circles, Polygons, and Solids

Time Frame: Approximately three weeks

Unit Description

This unit provides an examination of the measurements of perimeter and area associated with circles, polygons, and solids. Additional emphasis is also given to students' spatial understanding of 3-dimensional figures and the concepts of surface area and volume.

Student Understandings

Students measure the perimeter and area of plane figures and circles, and determine the surface area and volume associated with 3-dimensional figures. Students solve real-world measurement problems with or without the use of measurement tools. They are proficient in the use of both the standard and metric systems of linear measurement and can compare approximate relationships between systems.

Guiding Questions

1. Can students find the perimeters and areas of triangles, standard quadrilaterals, regular polygons, circles, as well as irregular figures for which sufficient information is provided?
2. Can students construct 2- and 3-dimensional figures when given the name, description, or attributes of the figure?
3. Can students use and determine which surface area or volume formula to use for rectangular solids, prisms, pyramids, cones, and spheres?
4. Can students compare approximate relationships between both the metric and standard system of measurement?
5. Can students solve problems and determine measurements involving arc lengths and areas of sectors of circles?

Unit 6 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Measurement	
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)
Grade 10	
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
Measurement	
7.	Find the volume and surface area of pyramids, spheres, and cones (M-3-H) (M-4-H)
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and *n*-gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
13.	Solve problems and determine and determine measurements involving chords, radii, arcs, angles, secants, and tangents of a circle (G-2-H)

Sample Activities

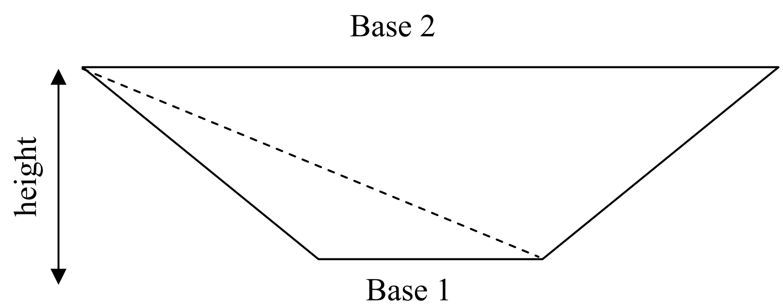
Activity 1: Review of Perimeter and Area Formulas for Polygons (GLEs: 9^{th} –21; 10^{th}–1, 12)

Materials List: paper, pencil, rulers, tape measures, meter sticks, 2-D items to measure perimeter and area, math textbook, Vocabulary Self-Awareness BLM

Provide copies of the Vocabulary Self-Awareness BLM to students. The goal of the *vocabulary self-awareness* (view literacy strategy descriptions) strategy is to make students comfortable with the vocabulary being used. In this particular use of the strategy, words and processes are being introduced using the BLM. Students complete a self-assessment of their comfort level with the terms and processes involved in the activity. This is a slight modification to the traditional *vocabulary self-awareness* strategy which typically just assesses students' comfort level with vocabulary. Have students complete Vocabulary Self-Awareness BLM, and use the results to target the vocabulary which students feel least comfortable with to help guide instruction throughout the activities that follow. After students have completed the BLM and a review of the terms has taken place, continue with the activities below. At the end of the activities, have students go back through the Self-Awareness BLM and have them assess their comfort level with the terms again. The goal is to have all students replace any O or – marks with + marks to indicate their understanding of what was taught throughout the lesson.

Review with students the formulas used in finding the perimeter and area of basic 2-D polygons, particularly special quadrilaterals and regular polygons. After discussing the formulas, provide 2-D items for students to determine the perimeter and area of through the use of measurement tools (using rulers, tape measures, and meter sticks) as well as pictures of figures with given measurements. This activity should include all of the following aspects:

- A thorough review of the metric and standard system of measurement for units associated with finding a length, including determining the most appropriate units and scales to use when solving measurement problems. Include measurement of objects to the nearest sixteenth of an inch and tenth of a centimeter and estimation of lengths of real objects and distances (such as estimating the perimeter of the classroom).
- Student investigations about the derivations of area formulas for quadrilaterals and how they are related to the area formula for a parallelogram, $A = bh$. For example:
  - The formula for the area of a triangle is derived from the fact that every triangle is one-half of the area of the parallelogram that encloses it; hence, $A = \frac{1}{2} bh$. Include problems dealing with determining the area of right triangles and a review of the Pythagorean Theorem.
  - The formula for the area of a trapezoid is derived from the fact that it is comprised of two triangles with the same heights but different bases. As shown in the figure below, the sum of the areas of the two triangles is: $A = \frac{1}{2} h$ (base 1) + $\frac{1}{2} h$ (base 2) or $A = \frac{1}{2} h$ (base 1 + base 2).



  - An introduction to the formula for the area of a rhombus which is based on the length of a rhombus' diagonals: $A = \frac{1}{2} d_1 d_2$
- Include applications of area and perimeter using the formulas discussed above in real-world problems. Use the math textbook as a resource for problems of this type.

Activity 2: Construction of 2-dimensional figures (GLEs: 10^{th}–9)

Materials List: paper, pencil, protractors, compasses, scissors

In this activity have students construct several 2-dimensional figures using straight edge, protractor, and compass when given the name of the figure and particular measurements or attributes for the construction. Use this activity as an opportunity to teach students the proper use of a compass and how to use a compass to make perpendicular segments and bisectors as well as how to make segment lengths for triangles (as in Task 1). Let students work in small groups to accomplish the following tasks:

Task 1: Draw a triangle with sides 5 cm, 6 cm, and 8 cm
Task 2: Draw a rectangle with sides 6 cm and 8 cm
Task 3: Draw a triangle with angles of 45 °, 55 °, and 80 °
Task 4: Draw a regular hexagon with sides of 5 cm

After students have drawn their figures, have them cut them out and compare their figures with those of the students in the other groups. Students should write down which figures are the same as (congruent to) those made by their peers and which are different. Discuss those which are congruent and which are not. Some of the things students should notice are as follows:

Task 1: All triangles should be congruent (this verifies the SSS Theorem for triangles).
Task 2: All rectangles should be congruent.
Task 3: Although all of the angles for the triangles should be congruent, the actual sizes of the triangles may be different from group to group. This also shows why there is no such thing as an AAA Theorem for triangle congruence.
Task 4: All hexagons should be congruent. As an extension activity, have students come up with a way to determine the area of this regular hexagon.

Activity 3: Volumes of Pyramids, Cones, and Spheres (GLE: 10^{th} –7)

Materials List: paper, pencil, volume model kit(s), rice or uncooked popcorn

The purpose of this activity is to have students discover the relationships between the volume of a rectangular prism and that of a pyramid, as well as to examine how the volumes of a cone and cylinder and a cone and a sphere are related. To examine these relationships, students should have access to volume model kits which can be purchased through many math catalogs.

Have students compare the volumes of a pyramid and a rectangular prism with the same base and height using a volume model kit. If enough model kits are available, have students work in groups of 3 to do this activity. If only one set is available, the teacher should perform a class demonstration to show the relationship between the pyramid and prism.

Have students fill the pyramid from the kit with rice or uncooked popcorn. Ask students to estimate how many times the pyramid would need to be filled and its contents poured into the rectangular prism in order for the prism to be filled. Develop the concept that the volume of a pyramid is one-third the volume of a rectangular prism with the same base and height, since it would take three fills of the pyramid to completely fill the prism. Write the formula for the volume of a pyramid and help students to understand that this formula is derived from its relationship with that of the corresponding rectangular prism.

Have students do the same thing with a cone and cylinder having the same base and height. Just as before, students should see that it would take 3 fills of the cone to fill the cylinder. Therefore, students should understand that the formula for the volume of a cone comes from its relationship with the cylinder.

Finally, ask students to estimate the relationship between the cone and the sphere which is also a part of the kit, and then test it. Since the cone must be filled twice before the sphere is filled, the sphere is twice as large as the cone's volume or $\frac{2}{3}$ the volume of the cylinder. Rather than just giving the formula for volume of a sphere to students, lead them through the algebra to generate the formula. Start with the formula for the volume of a cylinder: $\boldsymbol{V} = \pi \boldsymbol{r}^2 \boldsymbol{H}$. The formula for the volume of the cylinder is $\boldsymbol{V} = \frac{2}{3}\pi \boldsymbol{r}^2 \boldsymbol{H}$. Ask students how the diameter of the sphere compares to the height of the cylinder. They should note that the diameter is the same as the height which gives $\boldsymbol{V} = \frac{2}{3}\pi \boldsymbol{r}^2 \boldsymbol{D}$ or $\boldsymbol{V} = \frac{2}{3}\pi \boldsymbol{r}^2 (2\boldsymbol{r})$. This equates to $\boldsymbol{V} = \frac{4}{3}\pi \boldsymbol{r}^3$ cubic units.

Provide real-life applications in which students must find the volumes of cones, pyramids, prisms, and spheres.

Activity 4: Circumference and Area of Circles (GLEs: 10th–9, 13)

Materials List: paper, pencil, compasses, Internet

Begin this activity with a review of the relationship that exists between a circle's radius, its diameter, and its circumference. Write the following statement on the board and use it as an *opinionnaire* (view literacy strategy descriptions) to begin the class discussion: "Pi is the number 3.14…but no one knows where it comes from." Ask students to look at the statement and decide if they agree or disagree with the statement, and if they disagree, come up with reasons why they disagree. Get students to debate with one another and through the debate, lead students to understand that pi is actually the ratio of a circle's circumference to its diameter, and 3.14 is one approximation for its value (22/7 is another popular approximation as a fraction). Students should also understand that to find the circumference for a circle all that needs to be known is its diameter or radius, using the formulas $C = 2\pi r$ or πD. Provide students with sample problems where either the radius or diameter is given, and have students determine the circumference. Include problems where the circumference is known, and the students have to determine the circle's radius or diameter. Students should understand that these measurements are not exact when using the estimates for π. To express an exact answer, the symbol should be left in the answer such as 36π. Also, include opportunities for students to construct circles given the diameter or radius using a compass.

Next, discuss the formula for the area of a circle. The website http://www.worsleyschool.net/science/files/circle/area.html has an excellent explanation of the development of the formula for the area of a circle and is definitely worth the time needed to show students. Again, provide students with sample problems dealing with area of circles including real-life problems which deal with the topics of circumference and area.

Activity 5: Circumference and Arc Length (GLEs: 10th–13)

Materials List: paper, pencil, math textbook, Arc Length BLM

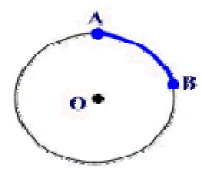
Extend students' knowledge of circles to include the definition of an arc of a circle and how to determine the length of an arc. First, define what a central angle is—an angle whose vertex is the center of a circle. Ask students if they remember how many degrees are in a circle. Students should know that there are 360°. Explain to students that sometimes the length of only a portion of a circle's circumference is needed. Present the term "arc" to the class, and explain that an arc is a fractional part of a circle's circumference.

For example, for a circle whose diameter is 3 meters, the circumference is found by multiplying (3 meters) by π which would be 3π meters or about 9.42 meters in length. Draw a circle with one 3 meter diameter. Ask students how many degrees would be in the central angle of an angle formed by the diameter. Students should realize that since the central angle is 180°, then this is essentially taking half the circumference, or $\frac{3}{2}\pi$ meters in this particular case. Next, have students determine what the arc length would be for this circle if the central angle is 90°. Again, students should realize that since the fractional part of the circle is $\frac{1}{4}$, one would then take $\frac{1}{4}$ of the circumference to find the arc length.

Ask students what process they would use to determine the arc length for a central angle of any degree measure. Let students figure out a process that works regardless of measure of the central angle. Students should realize that to determine an arc length, they simply have to determine the fractional part of the circle used by comparing the central angle to 360 and multiplying the fraction by the circle's circumference. Give time for students to think this process through rather than just telling them the process to be used. Students may benefit by using the proportion, $\frac{\text{central angle}}{360^0} = \frac{\text{arc length}}{\text{circumference}}$.

Provide additional examples to students using various angle measurements. When students are comfortable with determining arc length, provide students with copies of the Arc Length BLM. Let students work in pairs on the BLM, and afterwards fully discuss the concepts that are developed on the worksheet. Provide additional work for students on the topic of determining arc lengths by using the math textbook as a resource for additional problems.

Afterwards, have students create a *vocabulary cards* (view literacy strategy descriptions) for the terms arc length and central angle. An example is shown below:

Definition A fraction of the circle's circumference	*Characteristics* An arc is usually given as an angle, while an arc length is given as a fractional part of the whole circle, depending on the arc's angle.
Arc Length	
Examples If the whole circumference for a circle is 45 meters, then the arc length of a 30° angle of this whole circle would be 30/360 or 1/12 of 45 meters. 45/12 is about 3.75 meters.	*Illustrations* 

Allow time for students to review their vocabulary cards with a partner in preparation for quizzes and other class activities.

Activity 6: Sectors of Circles (GLEs: 10th - 13)

Materials List: paper, pencil, protractors, compasses, Sectors of Circles BLM

Just as an arc is a fractional part of a circle's circumference, a sector is a fractional part of a circle's area. Discuss with students the term *sector* and build on what students did with Activity 5 to help students to understand what a sector is and how to determine its area—a sector area is a fractional part of the circle's area. For example, suppose a circle's radius is 4 meters. The area of the circle would be 16π square meters or approximately 50.24 square meters. As they did in Activity 5, have students come up with a process to find the area of a sector given some central angle. Let them develop a procedure and then discuss their ideas before telling them the process. For example, if the area of a sector of the circle whose central angle was $30°$ was determined, since $30°$ is $\frac{1}{12}$ of the circle's area, the area of the sector would be $\frac{16\pi}{12}$ or about 4.2 square meters. This fractional part is proportional to the ratio ($\frac{x}{360}$) between the measure of the sectors central angle, *x*, and the number of degrees associated with a whole circle, $360°$. Provide additional examples for students, and once they are comfortable finding areas of sectors, have students work in pairs on Sectors of Circles BLM. Go over the BLM as a class. Afterwards, have students solve real-world problems that involve central angles and areas of sectors.

Activity 7: Surface Area (GLEs: 10th–7, 9)

Materials List: paper, pencil, scissors, rectangular cereal boxes (or equivalent), cylindrical boxes, cone-shaped coffee filters or drinking cups, pyramid-shaped model

Have students cut a 3-dimensional figure (such as cereal boxes, cylindrical oatmeal boxes, cone-shaped cups or filters, and pyramid-shaped models) to create a 2-dimensional figure that lies flat. Have students identify the shapes of the individual faces of the 3-dimensional figure using the 2-dimensional net. Discuss with students what surface area is, and help students understand the derivation of the surface area formulas for rectangular solids and cylinders (i.e., the formula of the surface area of a cylinder is based on the fact that it has two circular surfaces and the side of the cylinder is a rectangle, thus the surface area formula is: $S = 2\pi r^2 + 2\pi rh$). Include surface area formulas for rectangular solids, cylinders, pyramids, and cones.

Demonstrate to students how to use all of the formulas for surface area and apply them to real-world problems. As an extension, have students create a 3-dimensional figure that has specific dimensions. For example, have students make a cone that has the same height as a can of beans and the same radius as the can.

Activity 8: Which Shape Will Hold the Most Rice? (GLEs: 10th–9, 10)

Materials List: index cards, tape, rice, graduated cylinders

Provide each group of three students with three index cards, tape, rice, and a graduated cylinder. Have students fold the three cards and use tape to make each of these: a square prism, a regular hexagonal prism, and a circular cylinder (all without a top or bottom) using the shorter side of the card as the height of the prism or cylinder. Have students predict which container will hold the greatest amount of rice or if they all will hold the same. Have students record their prediction and justify their reasoning. Before continuing the activity, poll the class to see which figure it believes will hold the least amount of rice and which will hold the most, then discuss the predictions students made and their reasoning as a class.

Next, have students pour rice into each of the containers (using the desk as the bottom of each of the cylinders) and then use graduated cylinders to measure the amount of rice each was able to hold. Students might be surprised to find that the shape of the container will make a difference as to how much a container will hold—the circular cylinder will hold the most while the square prism will hold the least. Discuss the results as a class.

Have students take another index card and make another circular cylinder—this time using the longer side as the height of the cylinder. Ask students to venture a guess as to whether this circular cylinder will hold the same amount of rice as the previous one, and then determine the results. Students might be surprised to see how much more rice the first cylinder will hold even though both cylinders were made with the same index card.

Discuss mathematically in terms of the formula for the volume of a cylinder why the experiment had the results it did. The radius of the shorter cylinder was much larger than the radius of the taller version. Since the radius is "squared," this will result in the increased volume, even though the same index card is used. Have students show mathematically what the radius and height would be for each of the situations, and calculate the volumes for each cylinder using the formula for volume of a cylinder.

Activity 9: Volume Formulas (GLEs: 9th –21; 10th –7)

Materials List: paper, pencil, math textbook**,** tape

Begin the lesson by asking students to recall everything they know about the term volume. Students should understand that volume is a measurement of capacity—how much a container can hold. Volume tells us how many cubic (3-dimensional) units are contained within a given space.

Students should understand that capacity can be measured in cubic units (cubic inches, cubic feet, cubic yards, or cubic centimeters) or in units such as cups, quarts, liters, and gallons.

Make sure students understand how to compare capacity and volume units between systems using simple to use relationships. They should also have a feel for the relative size of each unit. For example, have students build models for cubic inches, or cubic feet as an outside project, and bring the models to class.

Students should know the relationship between the liter and the cubic centimeter—the liter is equivalent to 1000 cubic centimeters. Show students a cubic centimeter, and explain that a cubic centimeter actually holds 1 mL of liquid (make the connection to the fact that a milliliter is 1/1000 of a liter). All of this should be a review of previous work with volume from earlier grades.

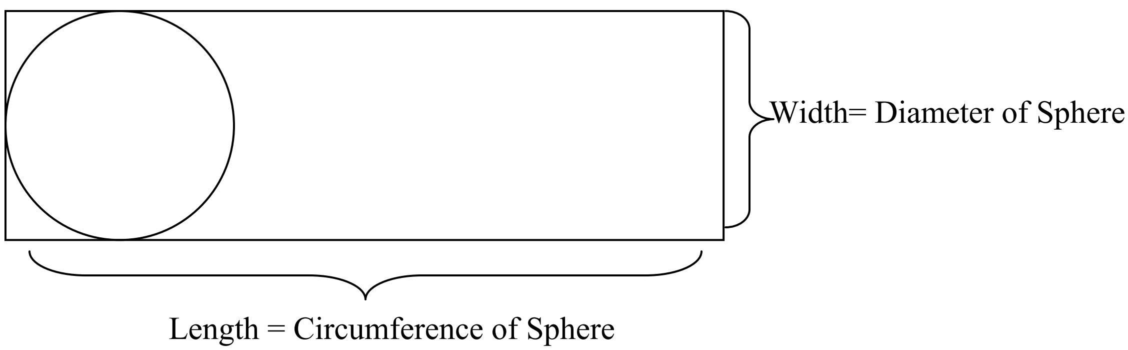
Review volume formulas for cylinders, cones, square pyramids, spheres and rectangular solids. Include work with real-world problems that apply these formulas. Use the math textbook as a resource for volume problems.

As an optional project for volume, have students construct an open cylinder (no top or bottom) using a single sheet of paper (and tape) that will hold one liter of rice. Have students construct the cylinder and then create a report detailing the process used to determine the dimensions needed to meet the specifications.

Activity 10: Surface Area and Volume of a Sphere (GLEs: 10th–7, 9)

Materials List: paper, pencil, various balls (baseball, basketball, etc.), wrapping paper, math textbook

This activity provides a concrete way to show students why the formula for the surface area of a sphere is $4\pi r^2$. Students should already understand that the surface area of an object can be represented by how much paper it would take to cover it. Ask them to then picture a sphere (a balloon or ball) and a piece of paper that is cut as wide as its diameter and as long as its circumference.



Have students form groups and provide students with different sized spheres (basketball, baseball, golf ball, or soccer ball) and wrapping paper to actually see if this will work. Students will see that this process would work for every sphere if it weren't for all the overlaps (which would fit into the gaps if you cut them out).

Connect the formula for the surface area of the paper with the dimensions for the rectangular piece of paper: Length x width = Circumference of sphere x Diameter of sphere. Since $C = 2\pi \times r$ and $d = 2r$, one gets $C \times d = 4\pi r^2$ = the surface area formula for a sphere.

Next, discuss with the students how to find the volume of a sphere, and have students find the volume and surface areas of the spheres they used in the wrapping activity. Finally, provide additional real-world application problems for students to solve dealing with the surface area and volume of spheres. Use the math textbook as a resource for problems of this type.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement.

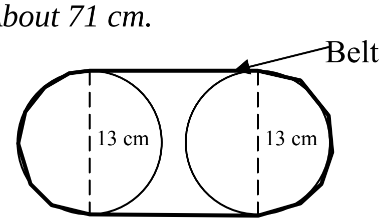
General Assessments

- The student will write a report on the history of measurement—how the different measurement units came into being.
- The student will take part in a measurement Olympiad and create a portfolio of his/her work.
- The student will calculate the density of an object by weighing the figure and by calculating its volume. Density is mass ÷ volume.

Activity-Specific Assessments

- Activity 1: The student will determine the area and perimeter of different 2-D shapes. The teacher will include problems involving finding shaded areas of a part of a complex figure or drawing.

- Activity 4: The student will solve problems associated with parts of a circle and their measurements in real world problem solving situations. For example: A belt is placed tightly along edge of two flywheels as shown in the picture below. Each of the flywheels has a diameter of 13 cm. The centers of the flywheels are 15 centimeters apart. Approximately how long is the belt?
  Solution: About 71 cm.



- Activity 9: The student will sketch diagrams and take appropriate measurements from actual objects needed to calculate volume. The student will label the sketches with measurements taken and then show the process used to calculate the answers. In addition, the student will solve real-world application problems involving volume.

Algebra I–Part 2
Unit 7: Patterns and Reasoning

Time Frame: Approximately two weeks

Unit Description

This unit introduces the concept of inductive reasoning and deductive reasoning in order to complete picture and number patterns and to find the rule for generating the *n*th term in a sequence of numbers.

Student Understandings

Students will represent number patterns using function tables, graphs, or equations. Students will match linear or nonlinear sets of data to a graph, determine the missing element in a number or shape pattern, and determine the nth element in a pattern.

Guiding Questions

1. Can students give examples of correct and incorrect usage of inductive reasoning?
2. Can students use inductive reasoning to find numerical and geometrical patterns?
3. Can students state the characteristics of a linear set of data and distinguish between those sets of data that are linear or non-linear in nature?
4. Can students determine the formula for finding the *n*th term in a linear data set?

Unit 7 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Algebra	
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Data Analysis, Probability, and Discrete Math	
29.	Create a scatter plot from a set of data and determine if the relationship is linear or non-linear (D-1-H) (D-6-H) (D-7-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

GLE #	GLE Text and Benchmarks
Grade 10	
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
2.	Predict the effect of operations on real numbers (e.g., the quotient of a positive number divided by a positive number less than 1 is greater than the original dividend) (N-3-H) (N-7-H)
4.	Use ratios and proportional reasoning to solve a variety of real-life problems including similar figures and scale drawings (N-6-H) (M-4-H)
Algebra	
5.	Write the equation of a line of best fit for a set of 2-variable real-life data presented in table or scatter plot form, with or without technology (A-2-H) (D-2-H)
Geometry	
17.	Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)
Data Analysis, Probability, and Discrete Math	
20.	Show or justify the correlation (match) between a linear or non-linear data set and a graph (D-2-H) (P-5-H)
22.	Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)
Patterns, Relations, and Functions	
26.	Generalize and represent patterns symbolically, with and without technology (P-1-H)
27.	Translate among tabular, graphical, and symbolic representations of patterns in real-life situations, with and without technology (P-2-H) (P-3-H) (A-3-H)

Sample Activities

Activity 1: Inductive vs. Deductive Reasoning (GLE: 10^{th}–17)

Materials List: paper, pencil, Inductive vs. Deductive Reasoning BLM

The process of reaching a conclusion based on information is called reasoning. The purpose of this activity is to provide students with the definition of inductive and deductive reasoning and to have students recognize when and how they are used in real-life situations.

Begin the activity by discussing inductive reasoning. Inductive reasoning is a type of reasoning that leads to a conclusion based on a pattern of examples or events and making an argument based upon that observation. For example, if one were to look at the list of numbers— 2, 9, 16, 23, and 30—most people would say that the next number is 37 since the pattern appears to add seven to each succeeding number. The conclusion is based upon reasoning—inductive reasoning. With inductive reasoning, one reasons from specific examples to make generalizations. Inductive reasoning can be the basis for a hypothesis. Making generalizations based upon only a few examples can, however, lead to faulty assumptions. For example, suppose the first nine players that a pitcher faced during a baseball game struck out. Can the assumption be made that the tenth batter will strike out, too? Can you say that the pitcher will never have anyone hit against him based upon the observation of the first nine batters striking out? In testing a general rule obtained through inductive reasoning, all it takes is one example that doesn't work to prove the rule false. A *counterexample* is an example which proves that an assumption cannot be true.

Next, discuss the term *deductive reasoning* and how it compares with inductive reasoning. Deductive reasoning is a process of reasoning logically from a set of given facts to produce a conclusion. For example, if a conditional statement is established as true, one can use deductive reasoning to prove a related statement.

Example: If you are in Louisiana, you are in the United States.

Since this conditional statement is true, the related statement can be made through the use of deductive reasoning.

Example: If you are not in the United States, then you cannot be in Louisiana.

Provide students with copies of the Inductive vs. Deductive Reasoning BLM. This BLM contains a variety of scenarios which involve inductive or deductive reasoning. Discuss the problems as a class. Students are required to identify which situations involve each type of reasoning, and explain when inductive reasoning might be used inappropriately (i.e., matters of coincidence rather than a true pattern).

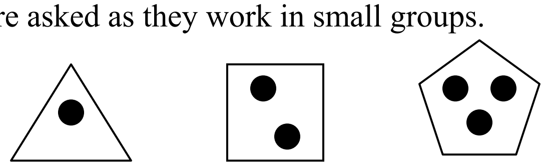
To complete this activity, have students write a math *learning log* (view literacy strategy descriptions) entry comparing and contrasting the differences between inductive and deductive reasoning. Have each student relate an incident in his/her own lives in which he/she has used each type of reasoning to make decisions or judgments. Follow up the writing by having students share their entries with their classmates to check for accuracy and logic.

Activity 2: Using Induction to Solve Picture Patterns (GLE: 10th–26)

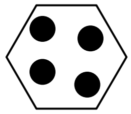
Materials List: paper, pencil, Example Patterns BLM, Using Inductive Reasoning BLM

The purpose of this activity is to have students use inductive reasoning to find the next picture in a pattern. Additionally, students will indicate verbally or in writing the process for generating the next item in the sequence. Make a transparency of the Example Patterns BLM which displays the two examples shown below in order to discuss them with students. Present the example problems to students, and have them answer the questions that are asked as they work in small groups.

Example 1:

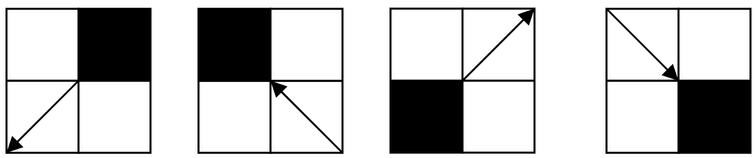


(a) Draw the next shape in the pattern

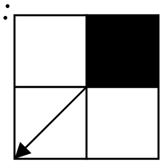
Solution: 

(b) Explain in words what the pattern is.
Solution: Each figure in the pattern has one more side associated with it, and the number of dots in the figure increases by one each time.

(c) Describe what the 50^{th} figure in the pattern would look like.
Solution: The 50^{th} figure would be a 52-sided regular polygon (52-gon) and would have 50 dots inside of it.

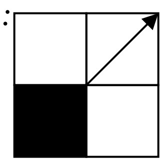
Example 2: 

(a) Draw the next figure in the pattern.
Solution:



(b) Describe the pattern in words.
Solution: *The figure gets rotated counter-clockwise 90°each time and the arrow is alternated with each turn from pointing away from the dark block to turning toward the dark block.*

(c) Draw what the 23^{rd} figure in the pattern would look like.
Solution:



After groups have had the opportunity to discuss the problems, go over the results as a class. Have students share their solutions and the reasoning they used as they came up with the solutions. Afterwards, provide students with copies of the Using Inductive Reasoning BLM. Allow students time to work on the BLM then discuss the results as a class.

Activity 3: Using Induction to Solve Arithmetic Number Patterns (GLE: 10^{th}–<u>26</u>)

Materials List: paper, pencil, Arithmetic Number Patterns BLM

The purpose of this activity is to allow students the opportunity to use inductive reasoning to find the next number in a pattern using arithmetic relationships. Problems include the use of rational, irrational, and integer numbers in patterns or other relationships (excluding geometric patterns where there is a common ratio which will be discussed in the next activity). Additionally, students should indicate verbally or in writing the process for generating the next term in a pattern. Present the following examples to students, and allow them the opportunity to think about the pattern. Have them write or explain in words the pattern they notice. Afterwards, discuss the results as a class:

Example 1: 1, 4, 9, 16, 25, 36, ____, _____.
Solution: 49, 64 (The numbers are perfect squares: $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$ *...; the answer can also be found by looking at the pattern resulting in the difference between each of the numbers listed.)*

Example 2: 1, 3, 7, 15, 31, 63, ____, _____.
Solution: 127, 255 (The differences between each set of two terms are 2, 4, 8, 16, 32, etc. The differences are doubled. The next difference should be 64, so $63 + 64 = 127$ *. The next difference would be 128, so* $128 + 127 = 255$ *.)*

Example 3: $\frac{1}{4}$, 1, $\frac{1}{2}$, $\frac{5}{4}$, $\frac{3}{4}$, $\frac{5}{2}$, 1, $\frac{7}{4}$, _______, _______
Solution: $\frac{5}{4}$ *, 2 (Each succeeding number in the pattern is alternating between adding* $\frac{3}{4}$ *to the number, and then subtracting the next number by* $\frac{1}{2}$ *.)*

Example 4: 1, 1, 2, 3, 5, 8, _______, _______
Solution: 13, 21 (Beginning with the third number in the list, each number is obtained by adding the previous two entries.)

After going over the examples as a class, provide copies of the Arithmetic Number Patterns BLM for students to work on in pairs. Discuss the results once students have completed the work.

Activity 4: Geometric Number Patterns (GLEs: 10th–1, 2, 4, 17, <u>26</u>)

Materials List: paper, pencil, math textbook, Geometric Patterns BLM

In this activity, patterns are introduced that are not arithmetic, but instead are geometric in nature. In a geometric pattern, a common number multiplies each successive element in the pattern. For example, in the pattern 2, 6, 18, 54 each successive number is being multiplied by the number 3. Numerically, notice that there is a common ratio associated with each pair of numbers: $6 \div 2 = 3$; $18 \div 6 = 3$; $54 \div 18 = 3$. Therefore, the next number in the pattern would be $54 \times 3 = 162$.

Sometimes the common ratio is not a whole number but a fraction. Discuss the following pattern: 12, 6, 3, $\frac{3}{2}, \frac{3}{4}$. The common ratio here is, $6 \div 12 = \frac{1}{2}$; $3 \div 6 = \frac{1}{2}$; $\frac{3}{2} \div 3 = \frac{1}{2}$. Each number is being multiplied by $\frac{1}{2}$. Therefore, the next number in the pattern will be $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

Present the examples shown below and have students attempt them in small groups before discussing the results as a class.

Example 1: $\sqrt{2},\ 2,\ 2\sqrt{2},\ 4,\ 4\sqrt{2}$, ______, ______

Solution: $8, 8\sqrt{2}$ *(Each number is being multiplied by* $\sqrt{2}$ *to get next value.)*

Example 2: $\frac{5}{3}, \frac{10}{9}, \frac{20}{27}, \frac{40}{81}$, _______, _______

Solution: $\frac{80}{243}, \frac{160}{729}$ *(Each number is being multiplied by* $\frac{2}{3}$ *.)*

Example 3: Find the tenth term for the pattern: -12, 24, -48, 96, …

Solution: The tenth term is 6144.

Example 4: Johnny created a pattern which started with 9 and in order to obtain each of the next terms, he divided the number before it by a fraction smaller than one. Will the terms after 9 be bigger or smaller than 9? Explain your answer.

Solution: The terms will be bigger than 9—when you divide by a fraction smaller than 1, you get a bigger number than you started with.

Next, provide students with copies of Geometric Patterns BLM. Allow students the opportunity to work on these problems in small groups, and then discuss the BLM as a class.

Afterwards, provide students with additional opportunities to become proficient at finding the next term when a geometric pattern exists. Mix up arithmetic, geometric, and other types of patterns to challenge the students to think and reason. Use the math textbook as a resource for additional problems of this type.

Activity 5: Patterns in Tables and Determining the Nth Term (GLEs: 9th–15; 10th–17, <u>26</u>, 27)

Materials List: paper, pencil, math textbook, Patterns in Tables BLM

This activity begins the development of strategies associated with finding the n^{th} term in a linear pattern. Provide students with copies of the Patterns in Tables BLM. Discuss the BLM with students before allowing them to work in small groups on the activity. Make sure that the students understand the directions and the tables that are presented on the BLM. In this activity, students are given an input/output table where the input is the term number and the output is the number associated with that position in a number pattern. Students use inductive reasoning to determine what is happening in the pattern and then write an expression for the nth term.

Once students have had the opportunity to discuss the problems presented on the BLM in their small groups, discuss the BLM as a class. Provide students with additional examples utilizing the textbook as a resource.

Activity 6: Making a Table to Determine the Nth Term (GLEs: 9th–15; 10th–17, 26, 27)

Materials List: paper, pencil, math textbook, Making a Table BLM

In Activity 5, students had to analyze a table to determine the relationship between the term number (input) and the term value (output) for a particular number. In this activity, students are given a pattern (linear in nature) and are asked to make a table to help them determine an expression for the n^{th} term. Provide students with copies of Making a Table BLM. Discuss the BLM with students to make sure they understand the directions, and relate this work to what was done in Activity 5. Allow students to work with a partner on the BLM, and then discuss the work as a class. Provide additional examples for students to become proficient at this skill, utilizing the textbook as a resource.

To complete this activity, have students form groups of four. Using a modified form of the *story chain* strategy (view literacy strategy descriptions) have the groups create a number pattern problem to be given to another group. In the typical use of a *story chain*, students write story problems on concepts being learned and then solve the problem. Typically, in a group of four students, one student writes the first sentence to the story problem; the second student writes the second sentence; the third student creates a question based on the first two sentences; and the fourth student solves the problem. In this particular use of the strategy, each group will create a problem to be solved by another group and then provide feedback on the answers.

Each group should perform the following tasks: (Examples of possible student work are provided below)

- Create a number pattern problem.
  - Example: 7, 9, 11, 13, …
- Make an input/output table relating the term number and its value.

Term # (n)	1	2	3	4
Output	7	9	11	13

- Come up with an expression which could be used to determine the *nth* term in the pattern that was created.
  - Ex: $7 + 2(n - 1)$ or $2n + 5$
- Once each group has created a problem and solution, it should exchange problems with another group. Each group then attempts the solution to the problem and is given feedback from the group that created the problem.

Once this process has concluded, the class should share any problems which were particularly interesting or unique.

Activity 7: Recognizing Linear Relationships in Table Formats (GLEs: 9th–15, 29, 37; 10th–5, 17, 20, 22, 26, 27)

Materials List: paper, pencil, graph paper, graphing calculators, math textbook**,** colored pencils

This activity is intended to be teacher-led. Present the following situation to students, and lead the students through each of the tasks. Allow students to work in small groups on each task as it is presented.

Problem: Joe was offered a 20-day job where he was given a choice of the following pay options.

Option 1: He would be given $50 the day he interviewed, and then when he returned to do the job, he would receive $145 per day for each day he worked during the 20 days.

Option 2: He would make 1¢ the first day of work, and each day he would be paid double the amount of money he received in payment on the day before.

Task 1: Which option seems like the best deal for Joe? Explain your reasoning.
Allow students to explain their reasoning for their choice. Answers may vary from student to student. Some students may think Option 1 is best based upon the higher starting pay. Many students may not realize that the nature of Option 2 will eventually result in a higher salary. This information will come out in the next few questions and tasks.

Task 2: Make a table for Option 1 relating the number of days worked and the total amount paid up until that point.

Table should look like the following:

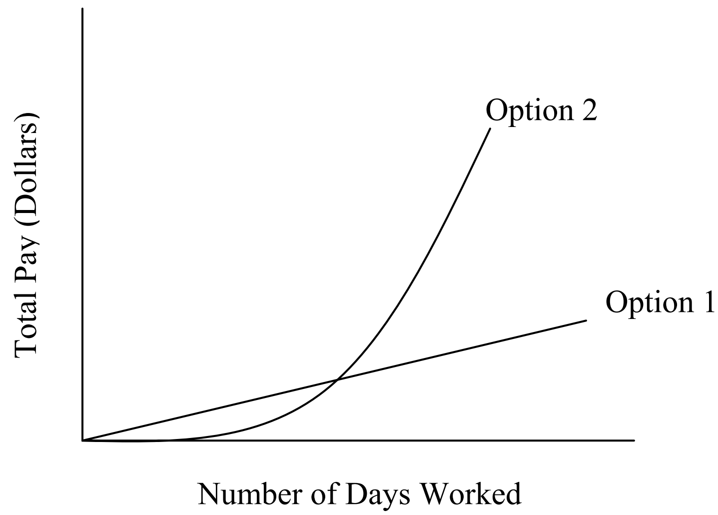
Days Worked	0	1	2	3	4	------	20
Total Money	$50	$195	$340	$485	$630	------	$2950

Task 3: Make a table for Option 2 relating the number of days worked and the total amount paid up until that point.

Table should look like the following:

Days Worked	0	1	2	3	4	------	20
Total Money	0	$0.01	$0.03	$0.07	$0.15	------	$10,485.75

Task 4: Create a graph for both options on a single sheet of graph paper. Choose an appropriate scale and label the graph. Use two different colors for each of the options.



Check student graphs. The basic graph should be as shown.

Task 5: Look at the graph you created from the data. Are any of the options linear? If so, which one?

Students should see that the graph of Option 1 is linear.

Task 6: Using the graph you created, find where the two graphs intersect and explain the significance of the point of intersection of the two graphs.

Students should see that the point where the two graphs intersect indicates when the two payment options are equal to one another. This point occurs at about the 10^{th} day of work, and the total amount paid is approximately $1500 dollars.

Task 7: Determine an equation that can be used to determine the total amount of money Joe gets paid for working *x* days for Option 1. To see if your equation is correct, let *x* be 20 to see if the total using your equation matches with the total you found in the table you created.

Solution: $y = 50 + 145x$ *could be used to determine the total amount paid, y, to Joe in dollars for working x days.*

Task 8: Explain the mathematical relationship which exists in the equation you wrote and the significance of the numbers that appear in the equation.

Students should see that the equation is linear in nature and that the slope represents the rate of pay per day, $145, and that the $50 is the y-intercept for the graph.

Refer back to the two tables that were created for the two payment options. Use this activity as an opportunity to emphasize to students that the ratios of the change in the *y* values and the change in *x* values remains constant in a set of linear data, such as in Option 1. In this case, the slope is constant. In a graph of non-linear data, this ratio changes, such as in Option 2. The data is non-linear and the slope is not constant. Provide additional opportunities for students to become proficient at identifying data as being linear or non-linear in nature. Use the math textbook as a resource for problems of this type.

Also, using graphing calculators, demonstrate for students how to draw scatter plots of data given in a table. Provide additional work on this skill by providing input/output tables, and have students determine if the scatter plots are linear or non-linear.

Activity 8: All About Patterns (GLEs: 9th—15, 29, 37; 10th—5, 17, 20, 22, 26, 27)

Materials List: paper, pencil, graphing calculators, All About Patterns BLM

In this activity, students are required to use everything they have learned about determining patterns including writing the n^{th} term, graphing scatter plots of the data, and then determining whether or not the data is linear or non-linear. Provide students with copies of All About Patterns BLM. Have students work in groups on this activity.

When students have completed the work, use the *professor know-it-all* strategy (view literacy strategy descriptions) to review what was learned. This strategy is typically used once coverage of content has been completed, which is the case here. In *professor know-it-all*, student groups will be called to the front of the class to answer questions from other groups related to determining patterns. When asked a question, the members of the group take turns answering the questions. Every five or ten minutes, change the groups and allow new "*professors*" to answer questions. Make sure students listen for accurate and logical answers from the "*professors*" to their questions about patterns.

Using the All About Patterns BLM, have students answer questions which arose during the activity. Moderate the question/answer session as needed. Be sure to include in the discussion the differences between the data sets to determine what makes a data set linear or not linear—in a linear set of data, the slope remains constant. Also, include the use of graphing calculators, and discuss finding a line of best fit.

Sample Assessments

General Guidelines

Performance assessments can be used to ascertain student achievement.

General Assessments

- The student will write a list of ways in which he/she uses inductive reasoning in everyday life.
- The student will use the writing process to explain in his/her own words how to determine a formula for a numerical pattern and how to use the formula to find the 50th term.
- The student will explain the difference between deductive and inductive reasoning in writing.

Activity-Specific Assessments

- Activity 2: Provide the student various patterns involving pictures or geometric figures and the student will do the following: explain the pattern in words; provide the next figure in the pattern; and extend the pattern to find the 60th term (this is arbitrary, of course—could be any term) in the pattern.

- Activity 3: Provide the student with arithmetic number patterns using fractions, decimals, integers, and irrational numbers, and the student will figure out the pattern, describe the pattern in words, and find the next term in the pattern.

- Activity 4: Provide the student with geometric number patterns, and the student will figure out the pattern, find the ratio that is being used to create each number, and then use that ratio to determine the next number in the pattern.

- Activity 6: The student will find the n^{th} term in a number pattern and explain how the formula for the n^{th} term was determined.

- Activities 7 and 8: Provide the student with a real-life situation in which a linear relationship exists, and where a non-linear relationship exists. The student will make a table of values which relates to each situation, draw a scatter plot of the data, determine if the relationship is linear or not, and if it is linear, come up with an equation for the line of best fit for the data. Also, the teacher will provide data sets and graphs and the student will match the data set with the graph that would match the data.

Algebra I–Part 2
Unit 8: Relations and Functions

Time Frame: Approximately two weeks

Unit Description

This unit investigates the role of relations and functions in the development of algebraic thinking and modeling.

Student Understandings

Students model the concepts of variables, functions, and relations as they occur in the real world and use the appropriate notation and terminology associated with functions. Students will determine if a relation is a function and determine the domain and range of functions. Students will also create scatter plots and determine a line of best fit for a set of data.

Guiding Questions

1. Can students explain what a function is?
2. Can students understand and apply the definition of a function in evaluating expressions (output rules) as to whether they are functions or not?
3. Can students apply the vertical line test to a graph to determine if it is a function or not?
4. Can students identify the domain and range for a given relation or function?
5. Can students create a scatter plot for a data set and determine a line of best fit?

Unit 8 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 9	
Algebra	
10.	Identify independent and dependent variables in a real-life algebraic relationship (A-1-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Patterns, Relations, and Functions	
35.	Determine if a relation is a function and use appropriate function notation (P-1-H)

36.	Identify the domain and range of functions (P-1-H)
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H), (P-5-H)
Grade 10	
Algebra	
5.	Write the equation of a line of best fit for a set of 2-variable real-life data presented in table or scatter plot form, with or without technology (A-2-H) (D-2-H)

Sample Activities

Activity 1: A Relation and its Domain and Range (GLEs: 9th–10, 36, 37)

Materials List: paper, pencil, a math textbook, Function Vocabulary BLM

Begin this activity by having students go through a *vocabulary self-awareness* (view literacy strategy descriptions) diagnostic consisting of the terms which will be introduced during this unit. Administer the Function Vocabulary BLM to students and have students assess their own level of understanding of the terms presented on the BLM. Pick up the results and use them to guide instruction through the unit, spending extra time on those topics students may be least knowledgeable about. Re-administer the BLM at the end of the unit to see student progress over the course of the unit.

After the administering of the BLM, ask students if they have ever heard of the terms *relation*, *domain*, and *range*. Based on the student responses to the question and the responses to the Function Vocabulary BLM, determine what needs to be clarified about the terms. Students should already understand that in real life there are all kinds of relationships between things. For example, the area of a square is related to the length of a side of the square; the price that a company asks for an item it is selling is related to the demand of the particular item; the temperature is related to the altitude at which the temperature is taken—the higher up you go, the colder the temperature. Students need to understand that any relationship that exists between two quantities is called a *relation*. This relation can be in the form of ordered pairs. For example, if the area of a square is related to the length of a side of a square, one could write the ordered pairs in the form: (Length of a side, Area of square). Therefore, some ordered pairs showing this relationship would be: (1,1); (2,4); (3,9); and $(4,16)$.

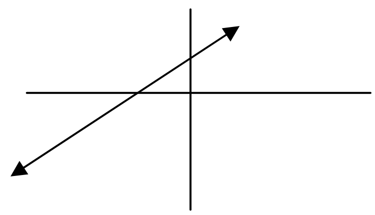
Next, discuss with students what the *domain* and *range* refer to for a group of ordered pairs. The *domain* refers to the set of *independent* or *x*-values for the relation while the range refers to the *dependent* or *y*-values of the relation. In the example showing the relationship between the length of a side and the area of a square, we have the following:

Domain = $\{1, 2, 3, 4\}$ Range = $\{1, 4, 9, 16\}$.

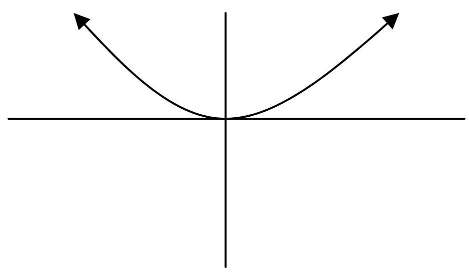
At this point, a review of the terms "dependent" and "independent" variable for a given relationship may be in order. For the example above, since the area of the square depends on the length of a side of the square, the length is said to be the independent variable while the area is referred to as the dependent variable.

Finally, discuss how to determine the domain and range for a graph. Some examples are shown below. We want students to see the domain as the "values that *x* can be" and the range as the "values that *y* can be." If there are no restrictions on any *x* or *y* values, then the domain and range are both "all real numbers" and can also be written using interval notation as $(-\infty, \infty)$. A discussion of interval notation may be needed.

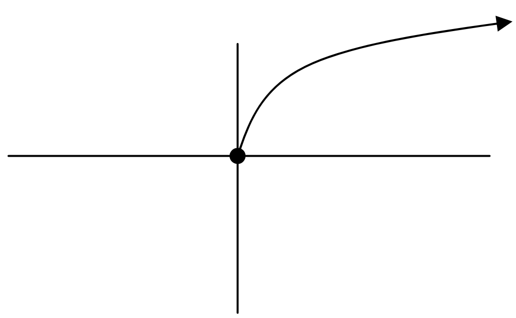
Example 1: Domain = all real numbers Range = all real numbers



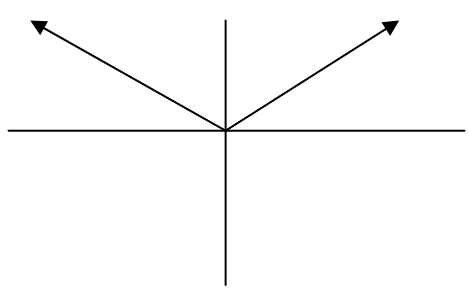
Example 2: Domain = all real numbers Range = $[0, \infty)$



Example 3: Domain = $[0, \infty)$ Range = $[0, \infty)$



Example 4: Domain = all real numbers Range: $[0, \infty)$



Provide students with additional opportunities to determine domain and range of ordered pairs and graphs of relations. Use the math textbook as a resource for additional problems.

Activity 2: Is It a Function? (GLEs: 9th–10, 15, 35, 36, 37)

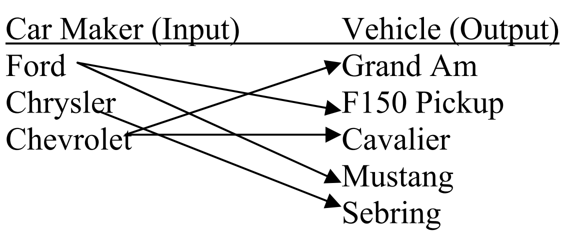
Materials List: paper, pencil, a math textbook, Is it a Function? BLM

Discuss with students a special type of relation that exists in math called a *function*. Explain to students that a function is a relation in which for each value of the independent variable there is exactly one value of the dependent variable that corresponds to it. Using the example provided before with the area of a square, explain how for each unique side length, there is only one area associated with a square with that given side length. Thus this relationship is a function, or in other words, "The area of a square is a function of its side length." Discuss other examples of functions with students and have them come up with additional real-life functions. Some are provided below:

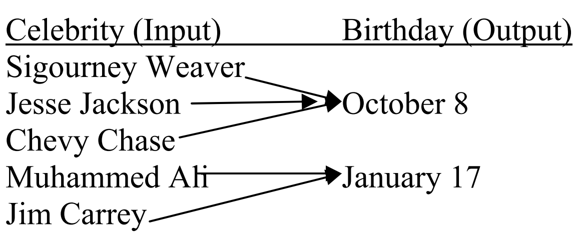
- For every numerical grade there is one unique letter grade associated with it; thus, the letter grade is a function of the numerical average.
- The cost for a water bill is a function of the amount of water used during the month.

Explain that for a relationship (or relation) to be a function, for each input there can only be one output value. If an input value is associated with two different output values, then it is not a function. Use the examples below to emphasize this concept.

- *Not a Function* (notice that some inputs have more than one output)

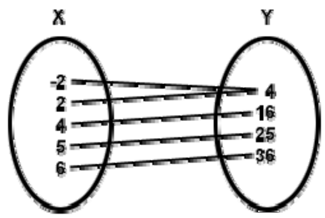


- *Function* (notice that each input corresponds to only one output, even though some inputs share the same output)



- <u>Function</u> (notice that each input corresponds to only one output, even though some inputs, -2 and 2, share the same output, 4)

<u>Input Output</u>



Provide additional examples as are needed for students to understand what constitutes a function when given a relation. Afterwards, provide students with copies of Is it a Function? BLM. Let students work in small groups to discuss the BLM and come up with their answers. Discuss the findings as a class. When discussing the BLM, have students identify the input as the independent variable and the output as the dependent variable. Students should also indicate the domain and range for each function identified. Follow this BLM with problems from a math textbook which deal with determining whether sets of data constitute a function.

Activity 3: Using Functional Notation (GLEs: 9th–<u>12</u>, <u>35</u>)

Materials List: paper, pencil, a math textbook

Demonstrate to students how to read and evaluate functional notation using a function such as $f(x) = 3x^2 + 2x - 4$. Explain to students that $f(x)$ is the functional value at *x* and can be thought of as the *y*-coordinate. Essentially, $f(x) = 3x^2 + 2x - 4$ is the same as the equation $y = 3x^2 + 2x - 4$.. Next, show students how to interpret and find functional values, such as $f(2)$. Help students understand the notation $f(2)$ means to substitute 2 for *x*. Demonstrate for students using $f(x)$ from above $f(2) = 3 * 2^2 + 2 * 2 - 4 = 12$. Therefore $f(2) = 12$ for this particular function. Make sure students understand that this is essentially the same thing as determining the *y*-value for a given *x*-value and that $f(2) = 12$ means that if the *x*-value is 2, the *y*-value is 12, which is the same as the ordered pair $(2,12)$. Provide additional opportunities for students to find values of given functions for various values of *x*. Use a math textbook as a resource for more problems of this type.

Activity 4: Evaluating Functions with Technology and the Vertical Line Test (GLEs: 9th–15, 35, 36)

Materials List: paper, pencil, graphing calculators

Have students use a graphing calculator to graph the following functions (one at a time):

$f(x) = x$

$f(x) = x^2$

$f(x) = 2x - 1$

Discuss the different aspects of each graph—which are linear or non-linear. For those that are linear, discuss what the slope of the graph is, and what the *x* and *y* intercepts are. At this time, explain to students that when given a graph, an easy test to determine whether it represents a function is to use the *vertical line test*. Explain what the vertical line test is, and how to use it. Explain why the vertical line test can help identify whether a graph is a function or not. If the same *x*-value (independent variable) is associated with two different *y*-values (dependent variable) then the graph is not a function and a vertical line would pass through more than one point on the given graph. Make sure students understand this connection. In the case of the three functions above, all will pass the vertical line test; thus, they are functions.

Next, have students graph $x = y^2$ using the graphing calculator. Students will need to know how to solve for *y* in $x = y^2$ in order to be able to enter this into the calculator. If students are not familiar with this skill, discuss it as a class. In order to input this equation into the calculator, it will have to be done using two separate equations: $y = \sqrt{x}$ and $y = -\sqrt{x}$. However, both equations should be graphed simultaneously on the calculator to be able to view the graph of the original equation, $x = y^2$. Ask students to determine whether this equation is a function using the vertical line test. Students should see that it does not pass the vertical line test, since a vertical line would intersect more than one point along its domain, thus $x = y^2$ is not a function.

Provide students with additional graphs and equations to determine whether the graph or equation is a function or not. Use a math textbook as a resource. Once the vertical line test is understood, revisit the functions and discuss how to determine the domain and range of each. Also, show students how to evaluate functions using the "value" function on a graphing calculator.

Complete this activity by having students write a math *learning log* (view literacy strategy descriptions) entry describing what the vertical line test is and how it works, along with an example of a graph which would not pass the vertical line test. Check student writings to make sure all students understand this concept, and provide appropriate feedback to students who may not fully understand how the vertical line test works.

Activity 5: Using Function Notation in Real-life Problems (GLEs: 9th–10, 12, 15, 35, 36, 37)

Materials List: paper, pencil, graphing calculators, a math textbook, Real Life Functions BLM

Provide students with copies of the Real Life Functions BLM. Allow students to work in pairs on the problems and questions related to a cost function. After students have had the opportunity to answer the questions in their pairs, allow each pair of students to get with another pair to compare answers before going over the results as a class. Provide additional problems involving real-world functions using a math textbook as a resource.

Activity 6: Finding the Line of Best Fit (GLEs: 9th– 15, 37; 10th–5)

Materials List: graph paper, pencil, Internet, spaghetti (uncooked to represent lines), graphing calculators, a math textbook

In this activity, students are exposed to scatter plots. Students will determine whether there is a positive, a negative, or no correlation in a data set and then find a line of best fit for a set of data. These topics will be introduced using several Internet websites that deal with these concepts.

First, go to http://www.regentsprep.org/Regents/math/data/scatter.htm which is a website that introduces scatter plots and the different types of correlation possible when a set of data is graphed. Discuss as a class. If no Internet connection is possible, print and make copies of the information for students.

Next, go to http://www.regentsprep.org/Regents/math/data/linefit.htm. This site contains detailed instructions for students to determine a line of best fit without a calculator using graph paper, pencil and a piece of spaghetti to represent the line.

Next, go to http://www.regentsprep.org/Regents/math/data/linefitcalc.htm. There are detailed instructions on how to use a graphing calculator to input data into lists and use the calculator to determine the TRUE line of best fit. Again, if no Internet connection is available, print the materials and provide copies to students.

Finally, after a full discussion of the material from the websites, provide students with sets of data (use a math textbook as a resource) and have students draw scatter plots using the paper/pencil method as well as graphing calculator technology. Have students determine a line of best fit for each data set, and write the equation using functional notation (if possible). Students should see that a relationship between two quantities could be displayed in different forms—in tables, in graphs, in words, and in symbols. These are all things students should already have experience with but are now using in the context of the terms *relation* and *function*

Sample Assessments

General Guidelines

Performance assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will look in newspapers to find examples of real-world functions and then write short reports about the examples of functions collected. The reports should include the identification of independent and dependent variables, with specific emphasis on describing them in terms of functions.
- The student will explain the difference between a relation and a function and why the vertical line test works when analyzing whether a graph is a function or not.
- The student will create portfolios containing samples of his/her activities.

Activity-Specific Assessments

- Activity 1: Provide the student with graphs and tables, and then have the student determine the domain and range of the relation.

- Activity 2: Provide the student with graphs and tables. Have the student determine whether or not the relation is a function and explain how he/she determined the answers.

- Activity 3: The student will evaluate functions using function notation.

- Activity 5: Provide the student with a real-world problem that uses function notation, and have the student answer questions related to the particular situation. For example, suppose $v(t) = -9.8t + 32$ which relates the velocity of a ball in meters/sec that is thrown at time, *t*, in seconds. The student will graph the function, give the domain and range, and find and explain the real-world meaning of the following: $v(0); v(2)$; for what *t* value will $v(t) = 0$.

  Solution: $v(0) = 32$ *which means that at the exact moment the ball is thrown, it is traveling at a velocity of 32 m/sec;* $v(2) = 12.4$ *which means that after 2 seconds, the ball has slowed down to a velocity of 12.4 m/s;* $v(t) = 0$ *approximately 3.27 sec after the ball is thrown.*